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PIKE'S
SYSTEM OF ARITHMETICK
ABRIDGED:

DESIGNED TO FACILITATE THE

STUDY OF THE SCIENCE OF NUMBERS.

COMPREHENDING

THE MOST PERSPICUOUS AND ACCURATE RULES,

ILLUSTRATED BY USEFUL EXAMPLES.

◆

TO WHICH ARE ADDED

APPROPRIATE QUESTIONS,

FOR THE EXAMINATION OF SCHOLARS;

AND

A Short System of Book-Keeping.

Second Edition, Revised, Corrected, and Improved.

BY DUDLEY LEAVITT,
Teacher of Mathematicks and Natural Philosophy.

Concord:
PUBLISHED BY JACOB B. MOORE.
1827.

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DISTRICT OF NEW-HAMPSHIRE, to wit.

District Clerk's Office.

L.S. BE IT REMEMBERED, that on the eighth day of April, A. D. 1826, and in the fiftieth year of the Independence of the United States of America, JACOB B. MOORE, of said district, hath deposited in this office the title of a book, the right whereof he claims as proprietor, in the words following, to wit:

"*Pike's System of Arithmetick Abridged: designed to facilitate the study of the science of numbers. Comprehending the most perspicuous and accurate rules, illustrated by useful examples. To which are added appropriate questions, for the examination of scholars; and a short system of Book-Keeping. By DUDLEY LEAVITT, Teacher of Mathematicks and Natural Philosophy.*"

In conformity to the act of the Congress of the United States, entitled "An act for the encouragement of learning, by securing the copies of maps, charts, and books, to the authors and proprietors of such copies, during the times therein mentioned;" and also to an act, entitled "an act supplementary to an act entitled an act for the encouragement of learning, by securing the copies of maps, charts and books, to the authors and proprietors of such copies, during the times therein mentioned, and extending the benefits thereof to the arts of designing, engraving and etching historical and other prints."

CHARLES W. CUTTER,
Clerk of the District of New-Hampshire.

A true copy of Record;—
Attest, CHARLES W. CUTTER, *Clerk.*

RECOMMENDATIONS.

[**Pike's Arithmetick** is universally acknowledged to be the most complete system ever published in the United States. It is the source, indeed, from which most of the common arithmeticks have been compiled. Several years of laborious study were devoted to the work by its author, and on its first appearance, it obtained a very high reputation. Others have built upon his foundation, and their works have been deservedly popular; but still, in the fulness and correctness of his rules, and the simplicity of their illustration, **Pike stands pre-eminent**.]

From Andrew Mack, Esq. Preceptor of the Academy at Haverhill, N. H.

Jacob B. Moore,

Sir,—I have examined your edition of Pike's Arithmetick, and am favorably impressed with the additions and improvements; and am of opinion that the work is as well adapted to use in our primary schools and academies, as any system of a similar kind that has come to my knowledge. Yours respectfully,

ANDREW MACK.

From Thomas Chadbourne, M. D. of Concord.

Pike's system of Arithmetick has long been classed among standard works, not only in primary schools, but in the higher seminaries.—The abridgment now published by Mr. Jacob B. Moore, embraces the most essential parts of the original work, condensed into a more portable form; and by the addition of new and practical illustrations, and appropriate questions, the present edition will be found eminently calculated both to lessen the labor of instruction, and facilitate the progress of the pupil. THO. CHADBOURNE.

Concord, June, 1826.

From Ephraim Kingsbury, Esq. Register of Deeds for Grafton County.

Dear Sir,—I have received from you by the hand of George Woodward, Esq. a copy of Pike's Arithmetick, abridged by Dudley Leavitt, Esq. and am decidedly of opinion, that it is best adapted for common schools of any Arithmetick within my knowledge. I am much pleased to see, what has long been wanting, a short system of Book-Keeping; and shall be pleased to see the Arithmetick introduced into our schools, and to meet with that patronage it justly deserves.

Very respectfully, Yours, EPHRAIM KINGSBURY.

Haverhill, June 15, 1826.

From Mr. Henry Fisk, Instructor of schools in Concord.

I have examined with much attention Pike's Arithmetick simplified by D. Leavitt, and think it the best calculated to facilitate the acquisition of this important branch of literature, of any treatise I have ever seen. I do cheerfully recommend it to the use of schools, thinking that greater utility may arise from it, than any other Arithmetick now in use. HENRY FISK.

Concord, July 5, 1826.

From Hon. Matthew Harvey, President of the Senate of New-Hampshire.

Hopkinton, December 4, 1826.

DEAR SIR,—After a careful examination of your improved edition of Pike's Arithmetick, I am decidedly of opinion there is no system, now in use, better arranged than this, for an easy acquirement of a correct and thorough knowledge of the principles of arithmetick. In all our schools, Leavitt's improvements must be convenient for the instructor, and useful to the scholar. With great respect, I am, Sir,

Your obedient servant, MATTHEW HARVEY.

Jacob B. Moore.

☞ The Hon. John Vose, Preceptor of Pembroke Academy, concurs in the above recommendation of Mr. Harvey.

From Amos J. Cook, Esq. Preceptor of Fryeburg Academy.

Fryeburg, Jan. 6, 1827.

DEAR SIR,—I have always entertained a great fondness for the System of Arithmetick, published more than forty years ago by Nicolas Pike, Esq. It is a work which brings to view great powers of mind, and displays deep mathematical research. This work has been abridged a number of times, and has appeared in different forms, and from different presses; yet still it has never in my opinion, promised so much usefulness to the common school and academy, as it now does from the hand of Mr. Dudley Leavitt, "Teacher of Mathematicks and Natural Philosophy." The judicious arrangement of the whole, and the notes and questions at the foot of its pages, much enhance the value of the publication. The paper, the type and the execution throughout reflect credit on the American Press.

I am, dear sir, affectionately your friend, AMOS J. COOK.

Jacob B. Moore.

From Benjamin Doe, Esq. Preceptor of Kingston Academy.

Kingston Academy, N. H. June 21, 1827.

Mr. Jacob B. Moore,—DEAR SIR,—The copy of your edition of Pike's Arithmetick, which you were pleased to present me, I have carefully examined. I have ever considered Pike's large work as the most complete system ever published. His abridgment has not, of late years, been so much used. I think, that the improvements which have been made in your edition of it, will render it much more valuable; and they will again be the means of bringing the work into more general use. The demonstrations and illustrations given in it, will make the science more attractive to the scholar. I wish you success in the undertaking. Yours respectfully, BENJAMIN DOE.

PUBLISHER'S NOTICE.

THE SYSTEM OF ARITHMETICK, of which this work is a new and careful abridgment, has been so long an inmate of our academies and higher seminaries of learning, that its merits are familiar to all. Though numerous treatises on the subject have successively appeared, since the work of PIKE was first published,* few have been able to sustain even an ephemeral reputation, excepting such as were built upon the labours of our author, yet simplified in some of the more intricate parts. Excellent as that work is acknowledged to have been, it had its defects; especially in its want of conformity to the federal notation, and of simplicity, and attraction to the scholar, in a few of the rules. An abridgment was some years since published, in which little else was done than to change the notation;—and for want of that conciseness in the fundamental rules, which some treatises of less real merit possessed, the book was superseded, and has been neglected in this section of the country, even by those who avow their preference for Pike's as a complete system.

At the suggestion of several experienced teachers, the Publisher was induced to put to press a NEW ABRIDGMENT of Pike's Arithmetick, embracing all such portions of the original work as should be necessary for the use of common schools, or of private learners; containing also, many new and practical illustrations of the more important rules. For this purpose, several improvements suggested by instructors of schools and academies have been incorporated in the work; and the whole has undergone the patient and careful revision of a gentleman well known to the publick, as a teacher of great merit both in Mathematicks and Natural Philosophy, and who has devoted many years to the instruction of youth of both sexes.

---

* In 1783.—NICOLAS PIKE was a native of Somersworth in New-Hampshire; was born Oct. 6, 1743; graduated at Harvard College, 1766; was employed a great portion of his life in instructing youth, and died at Newburyport, Ms. Dec. 9, 1819.

One great object in this abridgment has been to simplify the general rules, by placing before the scholar their constituent parts illustrated by plain and easy examples. In executing the work, nothing superfluous has been added, and nothing omitted that would contribute to perfect its design, and render it serviceable to youth. Those, however, who are in the habit of teaching superficially, with the view of flattering the pupil and the parent with the mistaken idea of extraordinary progress, may probably raise objections against the work, as containing too many things to be committed to memory—that they will burden and confuse the mind of the scholar. Such persons have yet to learn the capacity of the young mind. For, though it may be true that a mass of *complex* ideas crowded into the mind of a scholar would embarrass and perplex him—it is equally true, that in proportion to the number of simple propositions impressed upon the memory of a child, will be the progress of his understanding in strength and capacity.

In the arrangement of the present work, regard has been paid to the natural dependence of the several parts upon each other. Though in some instances, it differs from the common method, it is believed to be the more correct and useful. Some of the old and obsolete rules of Tare and Trett, &c. have been omitted, and the Duties and Custom-House Allowances of our own country substituted. Several rules, such as Position, Alligation, Permutation, &c. are inserted in this work, more for the purpose of gratifying the curiosity and exercising the mind of the scholar, than for their utility in business. The practical parts are those upon which the greatest attention has been and should be bestowed; and it is upon the improvements in these generally, as well as upon the established character of the author, that the publisher rests his belief of the merits of this edition.

April 14, 1826.

TABLE OF CONTENTS.

Page.
Numeration, 9
———— Table, 10
Notation by Roman Letters, 12
Addition, 13
———— Table, 14
Subtraction, 17
Multiplication, 20
———— Table, 21
Division, 27
———— Table, 28
Federal Money, 33
Table of American Coins, 33
Do. of Foreign Coins, 39
Compound Addition, 40
———— Subtraction, 52
Problems, resulting from a comparison of the preceding rules, 56
Reduction, 59
Vulgar Fractions, 70
Decimal Fractions, 89
Compound Multiplication, 99
———— Division, 105
Simple Interest, 111
Commission, 117
Buying or Selling Stocks, 118
Insurance, 119
Simple Interest by Decimals, 119
Compound Interest, 121
———— by Decimals, 123
Duodecimals, or Cross Multiplication, 124
The Single Rule of Three, 130

Page.
General Method of Making Taxes, 140
Rule of Three Direct, 142
———— Inverse, 143
Double Rule of Three, 143
Conjoined Proportion, 146
Practice, 148
Single Fellowship, 154
Double Fellowship, 156
Custom-House Allowances, 157
American Duties, 160
Discount, 161
Barter, 163
Loss and Gain, 165
Equation of Payments, 166
Involution, 168
Evolution, 168
Table of Powers, 169
Extraction of the Square Root, 170
Extraction of the Cube Root, 178
Rule for extracting the roots of all powers, 184
Alligation Medial, 185
Alligation Alternate, 186
Single Position, 191
Double Position, 192
Permutation, 194
Gauging, 195
Mechanical Powers—Of the Lever, 197
Of the Wheel and Axle, 197
Of the Screw, 198
Useful and Diverting Questions, 199

INDEX TO THE TABLES.

Addition, 14
Ale or Beer Measure, 50
Aliquot parts of money weight, &c. 112, 148, 149
American Coins, 33
Apothecaries' Weight, 44
Avoirdupois do. 43
Cloth Measure, 44
Decimals, 89
Decimal parts of a year, 120
Discount per cent. 149
Division, 28
Dry Measure, 51
Duodecimals, 124
English Money, 41
Federal Money, 33
Foreign Coins, 39
Interest by decimals, 119, 123
Land or Square Measure, 48
Long do. 45, 46
Money, 33, 41
Motion, 47
Multiplication, 21
Netation, 12
Numeration, 10
Pence and shillings, 41
Powers, 169
Solid Measure, 49
Square do. 48
Taxes, for making, 141
Time. 46
Troy Weight, 42
Wine Measure, 49

☞ Beside the tables above enumerated, the scholar will find in the CIPHERING-BOOK, published in connexion with this volume, a variety of very useful tables, and several not found in common arithmeticks.

EXPLANATION OF ARITHMETICAL SIGNS.

Signs.

$=$ Two parallel horizontal lines are the sign of equality. It shows that the number before, is equal in value to the number after it. *Example*, 1 dollar$=$100 cents, is read thus, 1 dollar is equal to 100 cents.

$+$ Two short lines, crossing each other at right angles, are the sign of Addition. It shows that numbers with this sign between them, are to be added together. *Example*, $5+7=12$, is read thus, 5 added to 7, or 5 *plus* 7, is equal to 12.

$-$ A short horizontal line is a sign of Subtraction. It shows that the number after it, is to be taken from the number before it. *Example*, $12-7=5$, is read thus, 12 less 7, or 12 *minus* 7, is equal to 5.

$\times$ Two short lines crossing each other in the form of an X, are the sign of Multiplication. It shows that the number before it, is to be multiplied by the number after it. *Example*, $6\times5=30$, is read thus, 6 multiplied by 5, is equal to 30.

$\div$ A short horizontal line between two points, is the sign of Division. It shows that the number before it, is to be divided by the number after it. *Example*, $30\div6=5$, is read thus, 30 divided by 6, is equal to 5.

$: \; : : \; :$ Four double points or colons are the sign of Proportion; and to show that numbers are proportional, they are written thus, $2 : 4 :: 8 : 16$, which are read, 2 is to 4 as 8 is to 16.

$\overline{\;\;}|^{2}$ This sign signifies the second power or square.

$\overline{\;\;}|^{3}$ This sign signifies the third power or cube.

$\sqrt[2]{}$ This sign, prefixed to any number, shows that the square root of the number is required.

$\sqrt[3]{}$ This sign, prefixed to any number, shows that the cube root of the number is required.

ARITHMETICK.

Arithmetick is the science of Numbers ;* and comprises the following principal rules, viz. I. **Notation**, or **Numeration** ; II. **Addition** ; III. **Subtraction** ; IV. **Multiplication** ; and V. **Division**. The four last are called *simple*, when the numbers are all of one denomination ; *compound*, when the numbers are of different denominations.

These five rules are called principal or fundamental, because the whole art of arithmetick is comprehended in their various operations.

◆

NUMERATION.

1. **Numeration** teaches us to read or write any sum or number, by means of the following ten characters, called figures :†

Cipher.	One.	Two.	Three.	Four.	Five.	Six.	Seven.	Eight.	Nine.
0,	1,	2,	3,	4,	5,	6,	7,	8,	9.

2. The value of a figure, when alone, is called its *simple* value, and is invariable. Figures have also a *local* value, which varies

---

* *Number* is either a unit, or a collection of units. A single thing is a unit, or one. One and one are two. One and two are three. And thus, by constant addition, all numbers are generated.

† These figures, which are of Arabic or Indian origin, were introduced into Europe by the Moors, about the year 1150; they were formerly all called ciphers, whence it came to pass that the art of arithmetick was called *ciphering*.

1. *What is Arithmetick ?*——2. *What are called its fundamental rules ?*——3. *What does numeration teach ?*——4. *How many figures are used to represent numbers ?*——5. *How do you determine their value ?*

according to the place in which they stand. In a combination of figures, reckoning from the right hand to the left, the figure at the right, or in the first place, represents its *simple* value; that in the second place, ten times the simple value; that in the third place, ten times the value of that in the second place; and so on, in a tenfold increase.

Thous.	Hund.	Tens.	Units.
4	4	4	4

EXAMPLE.—Write down the sum 4 4 4 4. The first figure at the right, in the place of units, has its simple value, or the same as if standing alone—*four*. The second, in the place of tens, signifies *four tens*, or *forty*. The third figure, in the place of hundreds, signifies *four hundred*, or ten times its value in the place of tens. The fourth figure is in the place of *thousands*, bearing ten times its value in the place immediately preceding.

3. A cipher 0 though of no signification itself, when placed on the right hand of figures, in whole numbers, increases their value in the same tenfold proportion. Thus 9, signifies only *nine*, its simple value. Place a cipher on the right, (90) it becomes *ninety*; and by placing two ciphers at the right, thus, (900) it becomes *nine hundred*.

NUMERATION TABLE.

1. Period of Units.
2. Do. of Thousands.
3. Do. of Millions.

3.				2.				1.			
Hundreds of Millions	*Tens of Millions*	*Millions*		*Hundreds of Thousands*	*Tens of Thousands*	*Thousands*		*Hundreds*	*Tens*	*Units*	
										9	nine units, or nine.
									9	8	nine tens, and eight units, or ninety-eight.
								9	8	7	nine hundreds, eight tens, and seven units, or [nine hund. 87.
						9	.	8	7	6	nine thousand, 876.
					9	8	.	7	6	5	ninety-eight thousand, 765.
				9	8	7	.	6	5	4	nine hundred eighty-seven thous. 654.
		9	.	8	7	6	.	5	4	3	nine millions, 876 thous. 543.
	9	8	.	7	6	5	.	4	3	2	ninety-eight millions, 765 thous. 432.
9	8	7	.	6	5	4	.	3	2	1	nine hundred 87 millions, 654 thous. 321.

Note.—Six places of figures, beginning on the right, are called a *period*; but they are commonly divided into *half periods* of three

---

6. As a combination of figures, how do you ascertain their value?——7. What is the nature of the cipher?——8. What is the period of Numeration?

figures each. This division enables us to read any number of figures as easily as we can read the first period.

Rule.—Commit the words at the head of the Table, viz. units, tens, hundreds, &c. to memory; then, to the simple value of each figure, join the name of its place, beginning at the left hand and reading towards the right. *More particularly*—1. Place a dot under the right hand figure of the 2d, 4th, 6th, 8th, &c. half periods, and the figure over such dot will, universally, have the name of thousands.—2. Place the figures 1, 2, 3, 4, &c. as indices, over the 2d, 3d, 4th, &c. period: These indices will then show the number of times the millions are involved—the figure under 1 bearing the name of millions, that under 2, the name of billions, or millions of millions;—that under 3, trillions, or millions of millions of millions.

EXAMPLE.

Sextillions.	Quintillions.	Quatrillions.	Trillions.	Billions.	Millions.	Units.
th. un.	th. un.	th. un.	th. un.	th. un.	th. un.	c.x.t.c.x.u.
6	5	4	3	2	1	
913,208.	000,341.	620,057.	219,356.	819,379.	120,406.	129,763
Thousands.	Thousands.	Thousands.	Thousands.	Thousands.	Thousands.	Thousands.

Note.—Billions is substituted for millions of millions; Trillions, for millions of millions of millions; Quatrillions, for millions of millions of millions of millions. Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, &c. answer to millions so often involved as their indices respectively denote.

The right hand figure of each half period has the place of units, of that half period; the middle one, that of tens, and the left hand one that of hundreds.

Application.—Let the scholar now read, or write down in words at length, the following numbers:—

8	437	709.040	3.476.194	7.184.397.647
17	3.010	879.096	84.094.007	49.163.189.186
129	76.506	4.091.875	690.748.591	500.098.422.700

Write down, in proper figures, the following numbers:—

Fifteen - - - - - - -

Two hundred and seventy-nine - -

---

9. *How are numbers commonly divided?*——10. *Of what use is this division?*——11. *What is the rule for numeration?*

Three thousand four hundred and three

Thirty-seven thousand five hundred and sixty-seven

Four hundred one thousand and twenty eight

Nine millions seventy-two thousand and two hundred

Fifty-five millions three hundred nine thousand and nine.

Eight hundred millions forty-four thousand and fifty-five

Two thousand five hundred and forty-three millions four hundred and thirty-one thousand seven hundred and two.

NOTATION BY ROMAN LETTERS.

Roman Notation is the method of representing numbers by Letters; and is now chiefly used to number the Chapters of books, &c. Seven letters are used for this purpose, viz. I, V, X, L, C, D, and M.—*I*, signifies 1; *V*, 5; *X*, 10; *L*, 50; *C*, 100; *D*, 500; and *M*, 1000.

TABLE.

I,	One.	XV,	Fifteen.	CC,	Two Hundred.
II,	Two.	XVI,	Sixteen.	CCC,	Three Hundred.
III,	Three.	XVII,	Seventeen.	CCCC,	Four Hundred.
IV,	Four.	XVIII,	Eighteen.	D, or IↃ,	Five Hundred.
V,	Five.	XIX,	Nineteen.	DC,	Six Hundred.
VI,	Six.	XX,	Twenty.	DCC,	Seven Hundred.
VII,	Seven.	XXX,	Thirty.	DCCC,	Eight Hundred.
VIII,	Eight.	XL,	Forty.	DCCCC,	Nine Hundred.
IX,	Nine.	L,	Fifty.	M, or CIↃ,	One Thousand.
X,	Ten.	LX,	Sixty.	IↃↃ,	Five Thousand.
XI,	Eleven.	LXX,	Seventy.	IↃↃↃ,	Fifty Thousand.*
XII,	Twelve	LXXX,	Eighty.	IↃↃↃIↃↃↃ,	Five Hund Thous.
XIII,	Thirteen.	XC,	Ninety.	MDCCCXXVII,	One Thous. Eight
XIV,	Fourteen.	C,	Hundred.		Hundred and Twenty-seven.

* Sometimes thousands are represented by drawing a line over the top of the numeral letter: thus $\overline{V}$ represents five thousand, $\overline{L}$ fifty thousand, $\overline{CC}$ two hundred thousand.

---

12. *What is Roman Notation?*——13. *What is its use?*——14. *How many letters are used for this purpose?*——15. *What number does each represent?*

RULES.

1. The number of a letter is doubled as often as it is repeated; thus, I, represents one; II, two; X, ten; XX, twenty; XXX, thirty.

2. A less literal number placed after a greater, augments the value of the greater; if put before, it diminishes it. Thus, VI, is 6; IV, is 4; XI, is 11; IX, is 9, &c.

* * *

ADDITION.

Addition is the putting together of two or more numbers, or sums, to make them one total or whole sum.

SIMPLE ADDITION

Is the adding of several numbers together, which are all of one sort, or kind; as, 7 pounds, 12 pounds, and 20 pounds, being added together, make a sum total, or aggregate, of 39 pounds.

RULE.

Place units under units, tens under tens, &c.: draw a line underneath, and begin with the units: After adding up every figure in that column, consider how many tens are contained in their sum, and placing the excess under the units, carrying so many as you have tens to the next column of tens: Proceed in the same manner through every column or row, and set down the whole amount of the last row.*

PROOF.

Begin at the top of each column, and add the figures downwards, in the same manner as they were added upwards, and, if it be right, this aggregate will be equal to the first amount. Or, cut off the upper line of figures, and find the amount of the rest; then if this amount and upper line, when added together, be equal to the sum total, the work is supposed to be right.

---

* This rule is founded on the known axiom, that "the whole is equal to the sum of all its parts." The method of placing the numbers, and *carrying for tens,* is evident from the nature of notation; for any other disposition of the numbers would alter their value; and carrying 1 for every 10, from an inferior to a superior denomination, is evidently right; because one unit in the latter case is equal to the value of ten units in the former.

---

16. *How are other numbers represented?*——17. *What is Addition?*——18. *What is Simple Addition?*——19. *Repeat the rule.*——20. *Why do you carry for ten, in adding simple numbers?*——21. *What is your method of proof?*

ADDITION TABLE.

[It is not necessary that this table should be committed to memory, so as to repeat it wholly out of the book. This would be indeed a tedious task. When the pupil can read the two first columns, viz. 2 and 6, 2 and 3, 2 and 7, &c., and cover the third, viz. 8, 5, 9, &c., and recite it readily, it will be sufficiently committed.]

2	and	6	are	8	5	and	9	are	14	8	and	11	are	19	11	and	6	are	17
2		3		5	5		13		18	8		7		15	11		3		14
2		7		9	5		7		12	8		13		21	11		8		19
2		4		6	5		3		8	8		3		11	11		4		15
2		9		11	5		5		10	8		8		16	11		12		23
2		5		7	5		11		16	8		5		13	11		7		18
2		8		10	5		8		13	8		10		18	11		2		13
2		12		14	5		6		11	8		6		14	11		9		20
2		2		4	5		2		7	8		2		10	11		13		24
2		10		12	5		12		17	8		12		20	11		5		16
2		11		13	5		4		9	8		9		17	11		11		22
2		13		15	5		10		15	8		4		12	11		10		21
3	and	9	are	12	6	and	13	are	19	9	and	6	are	15	12	and	14	are	26
3		7		10	6		7		13	9		3		12	12		6		18
3		5		8	6		3		9	9		7		16	12		3		15
3		8		11	6		8		14	9		13		22	12		10		22
3		6		9	6		5		11	9		10		19	12		7		19
3		4		7	6		12		18	9		7		16	12		9		21
3		2		5	6		4		10	9		5		14	12		5		17
3		10		13	6		10		16	9		2		11	12		2		14
3		3		6	6		2		8	9		12		21	12		8		20
3		12		15	6		6		12	9		8		17	12		13		25
3		11		14	6		9		15	9		4		13	12		4		16
3		13		16	6		11		17	9		11		20	12		11		23
4	and	13	are	17	7	and	9	are	16	10	and	9	are	19	13	and	2	are	15
4		5		9	7		13		20	10		13		23	13		7		20
4		11		15	7		3		10	10		7		17	13		10		23
4		8		12	7		7		14	10		11		21	13		3		16
4		2		6	7		11		18	10		3		13	13		8		21
4		6		10	7		5		12	10		10		20	13		5		18
4		3		7	7		8		15	10		4		14	13		13		26
4		12		16	7		2		9	10		6		16	13		9		22
4		7		11	7		6		13	10		12		22	13		4		17
4		10		14	7		12		19	10		5		15	13		12		25
4		9		13	7		4		11	10		2		12	13		6		19
4		4		8	7		10		17	10		8		18	13		11		24

EXAMPLES.

1. What is the amount of 3406, 7980, 345, and 27?

	Tens of Thous.	Thousands.	Hundreds.	Tens.	Units.
		3 .	4	0	6
		7 .	9	8	0
			3	4	5
				2	7
Total amount,	1	1 .	7	5	8
Amount with the upper line cut off,		8 .	3	5	2
Proof,	1	1 .	7	5	8

☞ Here we begin by writing down the several numbers, units under units, tens under tens, &c. Then draw a line under them.—We now commence adding at the foot of the right hand column, and say 7 and 5 five are 12, and 6 are 18. This exceeding ten, we write down the right hand figure 8 under the column of units, and carry one to the next column; and say, 1 and 2 are 3, and 4 are 7, and 8 are 15. We write down 5 at the foot of the column, and proceed to the next; 1 and 3 are 4, and 9 are 13, and 4 are 17. We write down in the same way the right hand figure, 7, under its column; and carrying 1 to the next, say 1 and 7 are 8, and 3 are 11. This being the last column, we write down the whole amount, 11, and find the sum total to be 11.758. The method of proof is sufficiently explained.

2.

389261
789794
849798
487697
999996
948219

3.

2136784
8297698
8297694
4876695
1234697
7092032

4.

37696941
49760823
45697615
82132435
48769206
48769209

5.

37856
975
1234
14
5612
2075
16287

6.

378269
202607
707
1246
2132
45178
10276

7.

141
2672
82971
34676
1459
427
12

8. What is the amount of three hundred and sixty-five, eight hundred and seven, five hundred and sixty, twenty-five, thirty-seven, and one hundred one? *Ans.* 1895.

9. The hind quarters of a cow weigh one hundred and three pounds each; the fore quarters weigh ninety-seven pounds each; the hide sixty-three pounds, and the tallow fifty-six pounds; what is the whole weight of the cow? *Ans.* 519 pounds.

10. A man has four farms; the first is worth two thousand seven hundred and twenty-five dollars; the second is worth three thousand eight hundred and nineteen dollars; the third is worth one thousand six hundred and ten dollars; the fourth is worth five hundred and twelve dollars; what are they all worth? *Ans.* 8.666 dollars.

11. A man possesses a tract of land, which contains forty-nine thousand eight hundred and thirty-five acres; suppose he had six tracts of equal dimensions, how many acres would the whole contain? *Ans.* 299.010.

12. What is the amount of three hundred, seventy five, two, forty-seven, thirty-three, nine thousand seven hundred and eighty-four, twenty thousand one hundred and fifty, seven hundred and sixty-five thousand and ninety-one, and one million seventy-five thousand and forty-seven? *Ans.* 1.870.529.

13. Add seventy-five millions nine hundred and sixty thousand eight hundred, two hundred and twenty-five thousand, and one hundred and forty together. *Ans.* 76.185.940.

14. What is the sum of four thousand and twenty-five, seventy-five thousand six hundred, eight hundred thousand four hundred and fifty, five millions three hundred and ten thousand, thirty millions seven hundred and twenty, and nine hundred fifty millions? *Ans.* 986.190.795.

15. What is the sum of one million five hundred thousand, three hundred and eleven thousand, ninety thousand six hundred and ten, fifty thousand and forty-five. *Ans.* 1.951.655

REMARKS.—As it is of great consequence in business to perform addition readily and exactly, the learner ought to practice it till it become quite familiar. If the learner can readily add any two digits, he will soon add a digit to a higher number with equal ease. It is only to add the unit place of that number to the digit, and if it exceed ten, it raises the amount accordingly. Thus, because 8 and 6 are 14, 48 and 6 are 54. It will be proper to mark down under the sums of each column, in a small hand, the figure *that is carried to* the next column. This prevents the trouble of going over the whole operation again, in case of interruption or mistake. If you want to keep the account clean, mark down the sum and figure you carry on a separate paper, and after revising them, transcribe the sum only. After some practice, we ought to acquire the habit of adding two or more figures at one glance. This is particularly useful when two figures which amount to 10, as 6 and 4, or 7 and 3, stand together in the column. Every operation in arithmetick ought to be revised to prevent mistakes; and as one is apt to fall into the same mistake if he revise it in the same manner he performed it, it is proper either to alter the order, or else to trace back the steps by which the operation advanced, which will lead us at last to the number we began with.

SUBTRACTION.

Subtraction teaches to take a less number from a greater, to find a third, shewing the inequality, excess or difference between the given numbers. The *greater* number is called the *Minuend*; the *lesser* number is called the *Subtrahend*. The difference between them, or what is left after subtraction is made, is called the *Remainder*.

SIMPLE SUBTRACTION

Teaches to find the difference between any two numbers, which are of a like kind.

RULE.

Place the larger number uppermost, and the less underneath, so that units may stand under units, tens under tens, &c.; then drawing a line underneath, begin with the units, and subtract the lower from the upper figure, and set down the remainder; but if the lower figure be greater than the upper, add ten to the upper, and subtract the lower figure therefrom: The difference

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22. *What is Subtraction?——23. What is the minuend?——24. What is the subtrahend?——25. What is the remainder?——26. How do you proceed in subtracting simple numbers?*

being set down, you must add one to the lower figure of the next column, for that which you borrowed; and thus proceed through the whole.

PROOF.

Add the remainder and the less number together; if the work be right, the amount will be equal to the greater number: *Or*, subtract the remainder from the greater sum, and the difference, will be equal to the less.

EXAMPLES.

From 3724 Minuend
Take 2583 Subtrahend
——
1141 Remainder
——
Proof 3724

☞ The operation of this example is very plain. The two sums being written down according to the rule, we draw a line underneath, and begin at the right hand figure, say—3 from 4 leaves 1, which we set down—in the next column, the subtrahend (8) being greater than the minuend, we add 10 to the upper figure, making it 12, and say, 8 from 12 there remain 4, which is written down. We now carry 1 to the next column, for that which we just borrowed, and say, 1 to five is 6, and 6 from 7 leaves 1, which we put down; and in the next column, taking 2 from 3, leaves 1, which we write down, and the work is done.—The method of proof will be extremely easy to the learner.

2.	3.	4.	5.
Miles.	Feet.	Cwt.	Dolls.
58934	879647	9187641	10000
6182	164343	91843	9999

6.	7.
1000200340000	100200300400400600700800900
2189918304	98076054032011023045067039

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27. *What is the method of proof?*

8. The Arabian or Indian method of notation was first known in England in 1150 how long is it since to the present time ?

9. What number must be added to 6892, so that the sum shall be 8265 ? Ans. 1373.

10. A gentleman purchased fifteen thousand eight hundred and forty acres of land ; he sold two thousand three hundred and fifty to one man ; four thousand five hundred to another ; and three thousand two hundred and twenty-five to a third ; how many acres had he left ? Ans. 5.765.

11. A man died, leaving an estate of 12.650 dollars, which he bequeathed as follows : 2.500 dollars to each of his three daughters, and the remainder to his son ; what was the son's share ? Ans. 5.150 dollars.

12. If the greater of two numbers be seven hundred and fifty millions, and the less five hundred and forty thousand ; what will be the remainder or difference ? Ans. 749.460.000.

13. A man's property is worth 10.650 dollars ; but he owes A 1.800 dollars, B 1.260 dollars, C 750 dollars, and D 500 dollars ;—what will remain after his debts are paid ? Ans. 6.340 dollars.

Remark.—At every stage of the scholar's progress, it would be well for him to review his previous studies—to fix in his memory the most important rules ; and endeavour to understand thoroughly the *reasonableness* and *accuracy* of the principles he is taught. The simple rules of arithmetick are easily acquired ; but if the scholar hurries over them, as is too often the case, without fully understanding their meaning and application, he will find his subsequent studies as much retarded, as he would find them aided, were he to become master of his subject as he goes along.—The instructor should question his pupils on every rule, and explain the *principles* of the rules, in every case, where a scholar desires it.

MULTIPLICATION.

Multiplication teaches how to increase the greater of two numbers given, as often as there are units in the less; performs the work of many additions in the most compendious manner; brings numbers of great denominations into small, as pounds into shillings, pence, or farthings, &c.; and, by knowing the value of one thing, we find the value of many.

There are three parts in Multiplication, viz.

The sum to be multiplied is called the *Multiplicand.*

The sum by which you multiply is called the *Multiplier.*

The result of the operation is called the *Product.*

The Multiplicand and Multiplier are likewise called both together *factors*, or that by which the operation is performed.

SIMPLE MULTIPLICATION

Is the multiplying of any two numbers together, without having regard to their signification; as, 7 times 8 are 56, &c.

RULE.

1. Place the Multiplier under the Multiplicand, so that units stand under units, and tens under tens, &c., and draw a line under them.

2. Beginning at the right hand, multiply each figure in the multiplicand by each in the multiplier, placing the first figure of every line directly under its respective multiplier, and to the product of the next figure carry one for every ten, as in addition.

3. Add the several products together, and their sum will be the total product required.

Proof.—Multiply the multiplier by the multiplicand.*

☞ *It is indispensably necessary that the Multiplication Table be committed perfectly to memory, before the scholar proceeds to the examples under the rules.*

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* The better way of proving multiplication is by Division. It may also be proved by casting out the 9's; but as the work will sometimes prove by that method, when in fact it is wrong, the rule is omitted.

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28. *What is multiplication?*——29. *What are the terms used in multiplication?*——30. *What is the multiplicand?*——31. *What the multiplier?*——32. *And what is the product?*——33. *What are the two first sometimes called?*——34. *What is Simple Multiplication?*——35. *What is the general rule for multiplying simple numbers?*——36. *How do you prove your sum to be right?*——37. *Repeat the Multiplication Table.*

MULTIPLICATION TABLE.

2	times	1	are	2
		2		4
		3		6
		4		8
		5		10
		6		12
		7		14
		8		16
		9		18
		10		20
		11		22
		12		24
3	times	1	are	3
		2		6
		3		9
		4		12
		5		15
		6		18
		7		21
		8		24
		9		27
		10		30
		11		33
		12		36
4	times	1	are	4
		2		8
		3		12
		4		16
		5		20
		6		24
		7		28
		8		32
		9		36
		10		40
		11		44
		12		48
5	times	1	are	5
		2		10
		3		15
		4		20
		5		25
		6		30
		7		35
		8		40
5	times	9	are	45
		10		50
		11		55
		12		60
6	times	1		6
		2		12
		3		18
		4		24
		5		30
		6		36
		7		42
		8		48
		9		54
		10		60
		11		66
		12		72
7	times	1	are	7
		2		14
		3		21
		4		28
		5		35
		6		42
		7		49
		8		56
		9		63
		10		70
		11		77
		12		84
8	times	1	are	8
		2		16
		3		24
		4		32
		5		40
		6		48
		7		56
		8		64
		9		72
		10		80
		11		88
		12		96
9	times	1	are	9
		2		18
		3		27
		4		36
9	times	5	are	45
		6		54
		7		63
		8		72
		9		81
		10		90
		11		99
		12		108
10	times	1		10
		2		20
		3		30
		4		40
		5		50
		6		60
		7		70
		8		80
		9		90
		10		100
		11		110
		12		120
11	times	1	are	11
		2		22
		3		33
		4		44
		5		55
		6		66
		7		77
		8		88
		9		99
		10		110
		11		121
		12		132
12	times	1	are	12
		2		24
		3		36
		4		48
		5		60
		6		72
		7		84
		8		96
		9		108
		10		120
		11		132
		12		144

EXAMPLES.

What is the product of 90631, multiplied by 8?

Multiplicand,	90631
Multiplier,	8
Product,	725.048

☞ Here, in the first place, we write down the multiplicand; then we write the multiplier under the unit figure of the multiplicand, and draw a line underneath. We then say, 8 times 1 are 8; as the product does not exceed nine, we write it underneath, we then say, 8 times 3 are 24, we write down the right hand figure 4, and reserve the left hand figure 2, to be added to the product of the next figure; we then say, 8 times 6 are 48 and two are 50, we write down the right hand figure 0, and reserve the left hand figure 5, to be added to the product of the next figure; we then say 8 times 0 is 0 and 5 are 5, and write down 5; we then say, 8 times 9 are 72: this being the last figure, we write down the whole product 72, and find the answer to be 725.048.

2.	3.	4.
769308	4980076	763896
3	4	5

5. What will be the product of 25.375, multiplied by 10?
6. Multiply 750.005 by 11.
7. Multiply 900.301.399 by 12.
8. Multiply 989.000 by 9.
9. Multiply 568.098 by 8.
10. Multiply 639.408 by 10.
11. Multiply 789.795 by 7.

CASE I.

When the multiplier consists of several figures.

RULE.—Write down the multiplicand, and under it the multiplier, units under units, tens under tens, &c. Then multiply by each significant figure in the multiplier separately, beginning with units, and writing the first figure of each different product directly under the figure by which you are multiplying. Add the several products together, and you have the sum total.

---

38. *What is your rule when the multiplier consists of several figures?*

EXAMPLES.

1. Multiply 7654 by 543.

	Proof by Multiplication.*
7654	543
543	7654
22962=3 times the multiplicand.	2172
30616=40 times do.	2715
38270=500 times do.	3258
	3801
4.156.122=543 times do.	
	4.156.122 Product..

☞ When the multiplier is a number consisting of several figures, after we have found the product of the multiplicand by the first figure of the multiplier, we suppose the multiplier to be divided into parts, and after the same manner, find the product of the multiplicand by the second figure of the multiplier; but as the figure by which we are multiplying, stands in the place of tens, the product must be ten times its number; and, therefore, the first figure in this product must must be written in the place of tens, or, which is the same, directly under the figure by which we are multiplying. And proceeding in the same manner with all the figures of the multiplier, separately, it is evident, we shall multiply all the parts of the multiplicand by all the parts of the multiplier; therefore the sum of these several products will be the whole product required.

☞ If there are ciphers between the significant figures of the multiplier, write them in a line with the product of the next significant figure, directly under their places in the multiplier. As in Example 2d.

2.	3.	4.
647906	46293845	91861284
4003	106	6875
1946718		
259162400		
2593567718		

---

* The reason of the method of proof by multiplication, depends upon this proposition, that if two numbers are to be multiplied together, either of them may be made the multiplier or the multiplicand, and the product will be the same. Example. 6×4=24, and 4×6=24.

CASE II.

When there are ciphers on the right hand of either the multiplicand or multiplier, or both.

Rule.—Neglect those ciphers ; then place the significant figures under one another, and multiply by them only ; add them together, as before directed, and place to the right hand as many ciphers as there are in both the factors.

EXAMPLES.

	1.	2.	3.
	67910	956700	930137000
	5600	320	9500
Prod.	380.296.000		

4. What is the value of a farm of 600 acres, at 20 dollars an acre ?

Ans. 12.000 dollars.

5. Multiply 50.750.000 by 75.000.

Ans. 3.806.250.000.000.

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39. *How do you proceed when any of the right hand figures of the multiplicand or multiplier are ciphers?*

CASE III.

To multiply by 10, 100, 1000, *&c.*

RULE.—Set down the multiplicand underneath, and join the ciphers in the multiplier to the right hand of them.*

EXAMPLES.

	1.	2.	3.	4.
	57935	84935	613975	8473965
	10	100	1000	10000
Prod.	579350			

CASE IV.

When the multiplier is a composite number, (or exactly equal to the product of any two figures in the multiplication table)—

RULE.—Multiply first by one of those figures, and that product by the other, the last product will be the answer.†

EXAMPLES.

	1.	2.	3.
Multiply	59375 by 35	39187 by 48	91132 by 56
	7		
7×5=35			
	415625		
	5		
Prod.	2078125		

---

* This is evident from the nature of numbers, since every cipher annexed to the right of a number increases the value of that number in a tenfold proportion.

† The reason of this rule is obvious; for if a given number be multiplied by any other, and that product again by another, the last product must be the same as if the given number were multiplied by the product of the two multipliers.

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40. *What is the rule when the multiplier is* 10, 100, 1000, *&c.?*—41. *When the multiplier is a composite number, what is your rule?*

CASE V.

To multiply by 9, 99, 999.

Rule.—Annex as many ciphers to the right of the multiplicand as there are figures in your multiplier, and from the number thus produced, subtract the given multiplicand, and the remainder will be the product.

EXAMPLES.

1. Multiply 5384976 by 9999.

53849760000
5384976
53844375024

☞ There being four 9s in the given multiplier, add four ciphers (0000) at the right hand; then write the multiplicand underneath, and subtract according to the rule.

2. Multiply 371967 by 999. Ans. 371595033.

◆

PROMISCUOUS EXERCISES.

1. What is the product of 237856, multiplied by 3729?
Ans. 887.099.968.

2. If 4 bushels of wheat make 1 barrel of flour, and the price of wheat be one dollar a bushel, what will 225 barrels of flour cost?

3. Multiply 308879 by twenty thousand five hundred and three.
Ans. 6.332.946.137.

4. Multiply 876956 by 990000.
Ans. 868.186.440.000

5. If a man rise an hour earlier every day, how much useful time will he gain for study or labour in 20 years, there being 365 days in a year?

6. What will be the total product of ninety-eight millions seven hundred sixty-three thousand five hundred and forty-two, multiplied by the same sum?
Ans. 9.754.237.228.385.764.

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42. *What is the rule for multiplying by* 9, 99, *&c.?*

DIVISION.

Division is the method of finding how many times a less number is contained in a greater; or dividing a quantity given, into any number of parts assigned; and is a concise way of performing several subtractions.

There are four parts to be noted in Division:

1. The *Dividend*, or number given to be divided.
2. The *Divisor*, or number given to divide by.
3. The *Quotient*, or answer to the question, which shows how often the divisor is contained in the dividend.
4. The *Remainder* (which is always less than the divisor, and of the same name with the dividend) is very uncertain, as there is sometimes a remainder, and sometimes none.

SIMPLE DIVISION,

Is the dividing of one number by another, without regard to their values; as 56, divided by 8, produces 7 in the quotient: That is, 8 is contained 7 times in 56.

GENERAL RULE.

1. Draw a curve line on each side of the dividend, and write the divisor at the left hand.
2. Take the same number of the first left hand figures in the dividend that there are in the divisor, if they be equal to, or greater than the divisor; but if they be less than the divisor, take one more; find the number of times the divisor is contained in them, and write a figure representing the number at the right hand of the dividend, which will be the first figure of the quotient.
3. Multiply the divisor by this quotient figure, and write the product under that part of the dividend taken.
4. Subtract this product from that part of the dividend taken, and bring down the next figure of the dividend, and place it at the right hand of the remainder; then find a quotient figure, multiply and subtract as before directed; proceed in the same manner until all the figures in the dividend are brought down and divided.
5. When a figure has been brought down and placed at the right hand of the remainder, if the number be less than the divisor write a cipher in the quotient, and bring down another figure.

---

43. *What is Division?*——44. *How many parts are there in Division, and what are they?*——45. *What is Simple Division?*——46. *What is your rule?*

Proof.—Multiply the divisor and quotient together, and add the remainder, if there be any, to the product: If the work be right, that sum will be equal to the dividend.

DIVISION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3		9	12	15	18	21	24	27	30	33	36
4			16	20	24	28	32	36	40	44	48
5				25	30	35	40	45	50	55	60
6					36	42	48	54	60	66	72
7						49	56	63	70	77	84
8							64	72	80	88	96
9								81	90	99	108
10									100	110	[illegible]0
11										121	132
12											144

To use the table.—Look for the divisor or number by which you wish to divide, in the left hand perpendicular column. Then trace the horizontal column in which the divisor stands, until you find the dividend or number into which you wish to divide, then trace that column to the top and you will find the quotient or number of times the divisor is contained in the dividend.—If you cannot find the exact number into which you wish to divide in the table, look for the next less one, and the difference between them will be what is over.

EXAMPLES.

1. How many times is 3 contained in 175817?

Divisor. Dividend. Quotient.
3) 1 7 5 8 1 7 (58605
1 5
———
2 5
2 4
———
1 8
1 8
———
1 7
1 5
———
2 Remainder.

☞ Here we first write down the dividend, and making a curve on each side, place the divisor (3) at the left hand. In this example, we see, that 3, the divisor, cannot be contained in 1, the first figure of the dividend; therefore we take two figures, (17) and inquire how often 3 is contained therein, which finding to be 5 times, we place the 5 in the quotient, and multiply the divisor by it, setting the first figure of the multiplication under the 7 in the dividend,

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47. *How do you prove your work to be right?*

dend, &c. We then subtract 15 from 17, and find a remainder of 2, to the right hand of which we bring down the next figure of the dividend, viz. 5; then, we inquire how often the divisor 3 is contained in 25, and finding it to be 8 times; we multiply by 8, and proceed as before, till we bring down the 1, when finding we cannot have the divisor in 1, we place 0 in the quotient, and bring down 7 to the 1, and proceed as at first.

PROOF.
58605 Quotient.
×3 Divisor+2
———
175817

Observe that the remainder 2, is here added in multiplying by 3.

Note.—When there is no remainder to a division, the quotient is the absolute and perfect answer to the question; but when there is a remainder, it may be observed, that it goes so much towards another time as it approaches the divisor; thus, if the remainder be half the divisor, it will go half of a time more, and so on; in order, therefore, to complete the quotient, put the last remainder to the end of it, above a line, and the divisor below it, as in Example 2. Hence the origin of vulgar fractions, which will be treated of hereafter.

The reason of the proof is plain; for, since the quotient is the number of times the dividend contains the divisor, the product of the quotient and divisor must, evidently, be equal to the dividend.

As the quotient and divisor are always multiplied during the operation, a simple method of proof is, by adding the several products and remainder, (if any) together as they stand. Thus in the above example—

1 prod. 15. . . .
2 prod . 24. . .
3 prod. . . 18 . .
4 prod. 15
Remainder 2
———
175817 Equal to dividend.

2.

29)153598(5296$\frac{14}{29}$
145
——
85
58
——
279
261
——
188
174
——
14

3.

6393)91876375(14150

---

48. *When there is no remainder, what is the quotient?*——49. *When there is a remainder, what is its nature?*——59. *What reason have you for your method of proof?*

CASE I.

SHORT DIVISION, *or when the divisor does not exceed* 12.

RULE 1. Find how often the divisor is contained in the first figure, or figures of the dividend, setting it under the dividend, and carrying the remainder, if any to the next figure, as so many tens.

2. Find how often the divisor is contained in this dividend, and set it down as before, continuing to do so, till all the figures in the dividend are used.

Note.—The work in Short Division is done *mentally*, that is, divided in the mind, and the result only written down; whereas in Long Division the operation is written at large.

EXAMPLES.

1.

4)924
———
231
———

☞ In this example, we write down the dividend, and draw a curve line at the left, and a straight line underneath. We then take the first figure of the dividend 9, and we find that 4 is contained in 9, 2 times; we write 2 under the 9, and then, mentally, multiply the divisor by it, and the product is 8, which we subtract from 9 and the remainder is 1; we then take the remainder [illegible] and the next figure of the dividend 2, and the number is 12; we find 4 is contained in 12, 3 times; we write 3 under the 2 and multiply the divisor by it, and the product is 12 which we subtract from 12, and nothing remains; we then take the next figure is the dividend 4, and find that 4 is contained in 4 once: we write 1 under the 4, and multiply the divisor by it, and the product is 4, which we subtract from 4, and nothing remains. All the figures in the dividend having been divided, we find that 4 is contained in 924, 231 times.

2.

2)71935
———

Quotient, 35967—1 remainder.
———

3.

12)1196437847536

CASE II.

When there is one cipher or more at the right hand of the divisor.

RULE.—It or they must be cut off; also cut off the same number of figures from the dividend, and then proceed as in case first: But the figures which were cut off from the dividend must be placed at the right hand of the remainder.

---

51. *What is Short Division?*——52. *How is it performed?*——53. *When there are ciphers at the right hand of the divisor, how do you proceed?*

EXAMPLES.

2.

65|00)3794326|75(58374
325

544
520

243
195

482
455

276
260

1675 Remainder.

3.

5193|000)8937643|893(

Note.—To divide by 10, 100, 1000, &c., cut off as many figures from the right hand of the dividend, as there are ciphers in the divisor; the left hand figures will be the quotient, and the right hand figures cut off will be the remainder.

CASE III.

When the divisor is such a number as any two or more figures in the table multiplied will make.

Rule.—Divide the dividend by one of these figures, and the quotient by the other; the last quotient will be the answer.

EXAMPLE.

1. What is the quotient of 196473 divided by 72 ?

1st method.	*2d method.*
9)196473	8)196473
8)21830	9)24559
Quotient, 2728—57 Remainder.	2728—57 Rem.

---

54. *What is your rule, when the divisor is the product of any two figures in the multiplication table?*

CASE IV.

To divide by fractions, or parts of a unit.

Rule.—If the numerator or upper figure, is a unit, multiply the given number by the denominator, or under figure, and the product will be the answer: But if the numerator is more than a unit, multiply the given number by the denominator, and divide the product by the numerator.

EXAMPLES.

1. Divide 848 by $\frac{1}{4}$
4=denominator.

Ans. 3392

2. Divide 496 by $\frac{3}{8}$
8=denom.

Numerator, 3)3968

$1322\frac{2}{3}$ *Ans.*

☞ *When the divisor is a whole number and a fraction—*

Multiply the whole number of the divisor by the denominator of the fractional part, and add the numerator to the product for a new divisor; then, multiply the dividend by the denominator of the fraction for a new dividend; lastly, divide the new dividend by the new divisor, and the quotient will be the answer.

EXAMPLES.

1. Divide 693 by $24\frac{3}{4}$

$24\frac{3}{4}$|693
4 4
99)2772(28 *Ans.*
198
792
792

☞ We multiply 24, the whole number of the divisor, by 4, the denominator of the fractional part, and add the numerator 3 to the product, and the sum is 99;—we then multiply the dividend 693 by the denominator of the fraction, and the product is 2772;—lastly, we divide 2772 by 99, and the quotient is 28; consequently $24\frac{3}{4}$ are contained in 693, 28 times.

2. Divide 6375 by $5\frac{1}{2}$ *Ans.* $1159\frac{1}{11}$.

3. In one rod there are $5\frac{1}{2}$ yards; how many rods are there in 1760 yards? *Ans.* 320.

4. What is the quotient of 10142, divided by $3\frac{2}{3}$? *Ans.* 2766.

---

55. *How do you divide by fractions?*——56. *How, when the divisor is a whole number and fraction?*

FEDERAL MONEY.

FEDERAL MONEY is the coin of the United States, established by Congress in 1786. The *Gold* Coins are the Eagle, Half Eagle, and Quarter Eagle. The *Silver* Coins are the Dollar, Half Dollar, Quarter Dollar, Dime and Half Dime. The *Copper* Coins are the Cent, and Half Cent. Mill is only imaginary, there being no Coin less than a half cent. The denominations of Federal Money are Eagles, Dollars, Dimes, Cents and Mills.

TABLE.*

10 Mills	make	Cent	c.	10= 1 cent
10 Cents		Dime	d.	100= 10= 1 dime
10 Dimes		Dollar $ or D.		1000= 100= 10= 1 dollar
10Dollars		Eagle	E.	10000=1000=100=10=eagle

In keeping accounts, and in reading Federal Money, eagles and dimes are not named; eagles being read tens, &c. of dollars;—and dimes, tens of cents. Dollars are separated from cents by a point or comma;—all the figures at the left hand of the point are dollars—the two first at the right hand are cents, and the third is mills. When there is a fourth figure, it represents tenths of mills—the fifth, hundredths of mills, &c. Thus, $25,7525 are read twenty-five dollars, seventy-five cents, two mills, and five tenths of a mill.

---

* By an act of Congress it was resolved, that there should be—

						Pure.	*Standard.*
Two Gold coins; viz.	1.	The Eagle	=	$10	weighing	247,5 grs.	270 grs.
	2.	The Half Eagle	=	5		123,75	135
Six Silver coins; viz.	1.	The Dollar	=	1		371,25	416
	2.	Half Dollar	=	,50 cts.		185,625	208
	3.	Quarter Dollar	=	,25		92,8125	104
	4.	Double Dime	=	,20		74,25	83,2
	5.	Dime	=	,10		37,125	41,6
	6.	Half Dime	=	,05		18,5625	20,8
Two Copper coins; viz.	1.	The Cent	=	,10 mills.		208	208
	2.	The Half Cent	=	,05		104	104

Any sum in Federal Money may be read either in the lowest denomination, or partly in the higher, and partly in the lowest; thus, $54,321 may be read 54321 mills, or 5432 cents 1 mill, or 543 dimes 2 cents 1 mill, or 5 eagles 4 dollars 3 dimes 2 cents 1 mill; all which denominations may be easily distinguished by the decimal point, thus—

E D. d. c. m.
5 , 4 , 3 , 2 , 1.

The method best adapted to practical purposes, and which has been sanctioned by a law of the United States, is the decimal form of expression by a decimal point, making the dollar the *money unit.* Dollars, therefore, will occupy the place of units, and the less denominations will be decimal parts of a dollar, and distinguished by the decimal point.

---

57. *What is Federal Money?*——58. *What are the different denominations?*——59. *What denominations are named in keeping accounts?*——60. *How are dollars separated from cents?*——61. *What are the figures at the left, and at the right of the point?*

Note 1. To write any number of cents less than ten—Write a figure representing the number of cents, then write a cipher at the left hand in the ten's place. Five cents are written thus—,05.

2. To write any number of mills less than ten, when there are no cents in the sum—Write a figure representing the number of mills, then write two ciphers at the left hand in the place of cents. Five mills are written thus—,005.

3. Parts or fractions of a cent are sometimes used instead of mills and tenths of a mill; as ,12½ cents for ,125.—,06¼ cents ,0625.

☞ As Federal money increases in a tenfold proportion, the rules for addition, subtraction, multiplication, and division of simple numbers, are applied to Federal Money. The rules for the operations in Decimals are strictly also applicable. [*See Decimal Fractions.*]

REDUCTION OF FEDERAL MONEY.

The method of changing or reducing the denominations of Federal Money is very simple, and needs perhaps no illustration, were the scholar always presumed to comprehend the preceding observations. In any case, the following concise rules will be easily understood.

RULES.	EXAMPLES.
	Given No. *Reduced.*
1. *To reduce dollars to cents:* Add two ciphers to the given number.	$25=2500 cents.
2. *To reduce dollars to mills:* Annex three ciphers to the given number.	$25=25000 mills.
3. *To reduce any sum of several figures to the lowest denomination mentioned:* Read the whole sum in the lowest denomination.	$12,24=1224 cents. $12,245=12245mills. ,125= 125mills.
4. *To reduce cents to dollars:* Cut off the two right hand figures of the given number, which will be cents—those remaining on the left are dollars.	Cents. c. 1275=$12,75
5. *To reduce mills to dollars:* Cut off the three right hand figures; those at the left are dollars—the two first on the right are cents, the third mills.	Mills. c. m. 75125=$75,12,5

---

62. How do you write a number of cents less than 10 *?——63. How is a less number of mills than* 10 *written ?——64. How are dollars reduced to cents and to mills ?——65. How do you reduce cents or mills to dollars ?*

REDUCTION OF OTHER CURRENCIES TO FEDERAL MONEY, &c.

CASE I.

To reduce Lawful Money, or the Currency of New-England, Virginia, Kentucky, and Tennessee, to Federal Money—the dollar being 6 shillings.

RULE.—As the value of a dollar is equal to three tenths of a pound, when pounds are given to be changed, annex three ciphers to the sum, and divide the whole by 3; the quotient is the answer in cents.

EXAMPLES.

1. Change £523 to Federal Money.

$3)523000$

$174333\frac{1}{3}$ cts. *Ans.* 1743 dolls. $33\frac{1}{3}$ cents.

2. Change £29 to Federal Money. *Ans.* $96 $66\frac{2}{3}$ cents.

When pounds and shillings are given, to the pounds annex half the number of shillings and two ciphers, if the number of shillings in the given sum be even; but if the number be odd, annex half the number, and then 5 and one cipher, and divide by 3; the quotient is the answer in cents.

EXAMPLES.

1. Change £59 18*s.* to Federal Money.

$3)59900$

$19966\frac{2}{3}$ cts. *Ans.* $199 $66\frac{2}{3}$ cents.

2. Change £93 13*s.* to Federal Money.

$3)93650$

$31216\frac{2}{3}$ cts. *Ans.* $312 $16\frac{2}{3}$ cents.

Change the following sums, viz.

	£.	*s.*		*dolls.*	*cts.*
3.	129	13	*Ans.*	432	$16\frac{2}{3}$
4.	63	15		212	50

When there are shillings, pence, &c. in the given sum, annex for the shillings as before directed, and to these add the farthings

---

66. *How is lawful money changed into the federal currency?——How, when there are different denominations, as pounds, shillings, pence, &c.?*

in the given pence and farthings, observing to increase their number by one when they exceed 12, and by two when they exceed 37, and divide as before.

EXAMPLES.

1. Change £21 8*s.* 4½*d.* to Federal Money.

3)21419
―――
7139⅔ cents.

4 is annexed to the pounds for half the shillings, and 19 for the farthings in 4½*d.* and excess of 12.

Ans. $71 39⅔ cents.

2. Change £721 8*s.* 11¼*d.* to Federal Money.

3)721497
―――
240499 cts.

In this example, 4 is annexed to the pounds for half the even shillings, and 47 for the farthings in 11¼*d.* and excess of 37, and then 5 is added to the figure next to half the shillings, making it 9 in place of 4 for the odd shilling.

Ans. $2404 99 cents.

3. Change £24 11*s.* 7¾*d.* to Federal Money.

Ans. $81 94.

4. Change £2001 1*s.* 3¼*d.* to Federal Money.

Ans. $6670 21⅔

CASE II.

To reduce New-York and North-Carolina Currency to Federal Money—the Dollar being 8 shillings.

RULE.—Add a cipher, and divide by 4.*

Note.—If there are shillings, pence, &c. given in any case, they must always be reduced to the decimal of a pound, and annexed to the given pounds before dividing by 3, &c.; and in all such cases three figures must be cut off at the right hand for decimals of a dollar.

EXAMPLES.

1. Reduce £44 New-York currency to Federal Money.

4)440
―――
110

Ans. $110.

2. Reduce £24 12*s.* to Federal Money. *Ans.* $61 50.

---

* As a dollar is equal to four-tenths of a pound, dividing pounds, or pounds and decimals of a pound, by 4, must, evidently, give dollars, cents and mills.

---

68. *What is your rule for changing New-York currency, &c. to Federal Money?*

CASE III.

To reduce Pennsylvania, New-Jersey, Delaware, Maryland, and Ohio Currency to Federal Money—the dollar being 7*s.* 6*d.*

Rule.—Multiply by 8, and divide the product by 3, the quotient will be the answer.*

EXAMPLE.

Reduce £243 New-Jersey to Federal Money.
8
3)1944
648=federal money. *Ans.* $648.

CASE IV.

To reduce South-Corolina and Georgia Currency to Federal Money—the dollar being 4*s.* 8*d.*

Rule.—Multiply by 30, and divide the product by 7, the quotient will be the answer.†

EXAMPLE.

Reduce £300 South-Carolina and Georgia to Federal Money.
300
30
7)9000
1285,71,4$\frac{2}{7}$=federal. *Ans.* $1285, 71,4$\frac{2}{7}$.

CASE V.

To change Sterling into Lawful Money—the dollar being 4*s.* 6*d.*

Rule.—Add ⅓ to the Sterling—the sum will be lawful.

---

* As a dollar is equal to three-eighths of a pound, dividing by ⅜, or which is the same, multiplying by 8 and dividing the quotient by 3, must give dollars, cents and mills.

† As a dollar is equal to seven-thirtieths of a pound, dividing by $\frac{7}{30}$, or which is the same, multiplying by 30 and dividing the quotient by 7, must give dollars, cents and mills.

69. *What is the rule for changing Pennsylvania, &c. to the same currency?*——70. *How is Georgia and South-Carolina currency changed?*——71. *How is sterling reduced to lawful money?*

EXAMPLE.

In £347 Sterling, how much Lawful?

3)347=sterling.
115 13 4 added.

462 13 4=lawful. *Ans.* £462 13 4.

CASE VI.

To Change Lawful to Sterling Money.

Rule.—From the Lawful subtract ¼, the remainder will be Sterling.

EXAMPLE.

Reduce £462 13 4 Lawful to Sterling.

4)462 13 4=lawful.
115 13 4 subtract.

347 00 0=sterling. *Ans.* £347.

CASE VII.

To change Canada and Nova-Scotia Currency to Federal Money—the dollar being 5 shillings.

Rule.—As the value of a dollar is equal to one-fourth of a pound, multiply the sum, when in pounds, by 4, for dollars.

When there are shillings, &c. reduce the given sum to pence, annex two ciphers, and divide by 60, for cents.

EXAMPLES.

1. Change £36 Canada Currency to Federal Money.

36
4

Ans. 144 dolls.

2. Change £528 12*s.* 6*d.* Canada Currency to Federal Money.

20

10572
12

6|0)1268700|0

211450 cts.

Or thus, 528
4

2112
10 shill.= 2
2*s.* 6*d.*= 0 50

2114 50
Ans. $2114 50 cts.

---

72. *Lawful to Sterling?*——73. *How is the currency of Canada and Nova-Scotia changed to federal money?*

CASE VIII.

To reduce English or Sterling Money to Federal Money.

RULE.—Multiply English money by 40,—divide the product by 9, and the quotient will be dollars, or dollars, cents and mills.*

EXAMPLE.

Reduce £100 English to Federal money.

£ 1 0 0
4 0

9)4 0 0 0

4 4 4 , 4 4 $\frac{4}{9}$ Ans.

◆

TABLE,

Shewing the rates at which the following foreign coins and currencies are estimated at the Custom-House of the United States,

Pound sterling of Great-Britain,	$4,44
Pound of Ireland,	4,10
Livre tournois of France,	,18$\frac{1}{2}$
Franc, do.	,18$\frac{3}{4}$
Silver Rouble of Russia,	,75
Florin or Guilder of the United Netherlands,	,40
Mark Banco of Hamburg,	,33$\frac{1}{3}$
Rix Dollar of Sweden and Denmark,	1,00
Real of Plate of Spain,	,10
Real of Vellon of do.	,05
Milrea of Portugal,	1,24
Tale of China,	1,48
Pagoda of India,	1,84
Rupee of Bengal,	,50

NOTE.—Any number of either of the above coins or currencies may be easily reduced to Federal money, by multiplying the value of one in cents by the given number, and the product will be cents.

---

* As a dollar is equal to nine fortieths of a pound, therefore multiplying by 40, and dividing the product by 9, must give dollars, cents and mills.

74. *How is sterling money reduced to federal ?*——75. *What is the value in federal money of the pound sterling of Great-Britain ?*——76. *Of the Irish pound ?*——77. *Of the Livre tournois of France ?*——78. *Of the silver rouble of Russia ?*——79. *Of the tale of China ?*——80. *And of the rupee of Bengal ?*

COMPOUND ADDITION.

Compound Addition is the adding together of several numbers having different denominations, as, pounds, shillings, pence, &c. ; tons, hundreds, quarters, &c.

RULE.

1. Place the numbers, so that those of the same denomination may stand directly under each other.

2. Add the first column or denomination together as in whole numbers ; then divide the sum by as many of the same denomination as make one of the next greater, setting down the remainder under the column added ; carry the quotient to the next superior denomination, continuing the same to the last column, which add as in simple addition.*

Proof.—The same as in simple addition.

1. MONEY.

Table of Federal Money.....(See page 33.)

EXAMPLES.

1.

E.	*D.*	*d.*	*c.*	*m.*
7	3	8	9	5
	2	1	2	5
9	0	0	5	
	3	6	2	5
7	1	4	0	8
24	1	1	0	3

2.

D.	*c.*	*m.*
375		
29	18	
7	12	5
199	18	7
30	01	

---

* The reason of this rule is evident from what has been said in addition of simple numbers: for in addition of money, as 1 in the pence is equal to 4 in the farthings ; 1 in the shillings, to 12 in the pence ; and 1 in the pounds, to 20 in the shillings ;—therefore, carrying as directed, is arranging the money arising from each column, properly, in the scale of denominations: and this reasoning will hold good in the addition of compound numbers of any denomination whatever.

81. *What is Compound Addition ?* ——82. *What is the rule ?*——83. *What the method of proof ?*——84. *How many mills, &c. are there in an eagle ?*

Table of English Money.

4 Farthings	make one	Penny,	*marked, qrs.* *d.*
12 Pence		Shilling,	*s.*
20 Shillings		Pound,	£.

Note.....¼=1 farthing, or quarter of any thing.
½=2 farthings, or half of any thing.
¾=3 farthings, or three quarters of any thing.

PENCE AND SHILLING TABLE.

20 pence make	*s.*1	*d.*8	100	*s.*8	*d.*4	60	£3	*s.*0
30	2	6	110	9	2	70	3	10
40	3	4	120	10	0	80	4	0
50	4	2	240	20	0	90	4	10
60	5	0	20 shill. make	£1	*s.*0	100	5	0
70	5	10	30	1	10	110	5	10
80	6	8	40	2	0	120	6	0
90	7	6	50	2	10	130	6	10

EXAMPLES.

1.

	£.	*s.*	*d.*
	47	17	11
	24	13	9
	36	10	6
	21	15	7
	17	14	6
	15	11	5
	10	10	2
Ans.	174	13	10
	126	15	11
Prf.	174	13	10

☞ In this example, we begin by placing the numbers of one denomination under each other, that is to say,—pounds under pounds, shillings under shillings, and pence under pence. We then begin with the least denomination, which is pence, and find the amount to be 46, which we bring into shillings by dividing them by 12—we write the remainder 10 under the column, and carry the quotient 3 to the column of shillings; we then find the amount of the column of shillings to be 93, which we bring into pounds by dividing them by 20; we write the remainder 13 under the column of shillings, and carry the quotient 4 to the pounds; we then find the amount of the column of pounds to be 174, which we write under the column of pounds, and find the answer to be 174£. 13s. 10d.

---

85. *What is a farthing?*——86. *Repeat the table of English Money.*——87. *How many pence and farthings are there in a pound?*

2.

£	s.	d.	qrs.
847	11	11	1
491	19	6	1
59	6	10	0
747	16	1	2

3.

£	s.	d.	qrs.
915	10	10	2
64	9	9	1
419	2	10	2
491	17	6	1
762	17	6	1
9	16	11	3

2. TROY WEIGHT.*

☞ By this weight are weighed Gold, Silver, Jewels, Electuaries, and all Liquors.

Table.

24 Grains	*grs.*	MAKE ONE	Pennyweight, marked,	*pwt.*
20 Pennyweights			Ounce,	*oz.*
12 Ounces			Pound,	℔ or *lb.*

EXAMPLES.

1.

℔.	oz.	pwt.
767	10	17
39	6	9
417	11	16
935	9	8

2.

℔.	oz.	pwt.	grs.
649	11	19	20
32	9	6	5
841	10	11	20
473	9	4	9

3.

℔.	oz.	pwt.	grs.
859	6	15	20
437	10	17	22
642	3	7	
738	9	4	23

---

* The original of all weights used in England, was a grain or kernel of wheat, gathered out of the middle of the ear, and, being well dried, 32 of them were to make one pennyweight, 20 pennyweights one ounce, and 12 ounces one pound. But in latter times it was thought sufficient to divide the same pennyweight into 24 equal parts, still called grains, being the least weight now in common use; and from thence the rest are computed.

88. What is Troy Weight?——*89. What are its denominations?*

Note.—Gold is tried by fire, and reckoned in *carats*, or the 24th part of any quantity. Such gold as will abide the fire without loss is accounted 24 carats fine ; if it lose 2 carats, it is called 22 carats fine, &c. A pound of silver which loses nothing in trial, is 12 ounces fine. *Alloy* is the base metal mixed with gold or silver, which abates its fineness. ☞ 175 Troy oz. are equal to 192 Avoirdupois oz., and 175 lbs. Troy are equal to 144 lbs. Avoirdupois. 1 lb. Troy=5760 grains, and 1 lb. Avoirdupois=7000 grains.

3. AVOIRDUPOIS WEIGHT.

☞ By Avoirdupois are weighed all coarse and drossy goods, groceries, bread, tallow, hay, leather, and all metals, except gold and silver.

Table.

16 Drams,	*dr.*	MAKE ONE	Ounce, marked	*oz.*
16 Ounces			Pound,	*lb.*
28 Pounds			Quarter of a hund. wt.	*qr.*
4 Quarters			Hun. weight, 112 lbs.	*Cwt.*
20 Hundred Weight			Ton,	*T.*

EXAMPLES.

1.

lb.	oz.	dr.
19	13	12
21	9	6
4	15	15
22	10	5

2.

T.	cwt.	qrs.	lb.
59	13	2	17
6	17	1	21
45	11	3	25
57	16	2	19

3.

T.	cwt.	qrs.	lb.	oz.	dr.
91	17	2	25	13	15
19	9	0	17	10	12
14	13	2	0	9	11
47	11	3	19	14	0

Note.—In Avoirdupois Weight, several other denominations are used in particular goods, viz.

A bbl. of Pot Ashes,	=	200 lbs.	144 dozen,	=	1 great gross.
A bbl. of Pork,	=	220 lbs.	20 particular things,	=	1 score.
A bbl. of Beef,	=	220 lbs.	5 do. do.	=	1 tally.
A quintal of Fish.	=	112 lbs.	24 sheets of paper,	=	1 quire.
12 particular things,	=	1 dozen.	20 quires,	=	1 ream.
12 dozen,	=	1 gross.			

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90. *What is a Carat?*——91. *What is the difference between Troy and Avoirdupois?*——92. *What is the use of Avoirdupois?*——93. *Repeat the table.*——94. *What other denominations are used in Avoirdupois?*

4. APOTHECARIES' WEIGHT.

☞ By this weight, apothecaries and physicians mix their medicines —but they buy and sell their commodities by avoirdupois.

Table.

20 Grains,	*grs.*	MAKE ONE	Scruple, *marked*,	℈
3 Scruples,			Dram,	ʒ
8 Drams,			Ounce,	℥
12 Ounces,			Pound,	℔

EXAMPLES.

1.

ʒ	℈	gr.
9	1	17
3	2	14
6	1	17
4	0	6
3	1	7
5	2	0

2.

℥	ʒ	℈	gr.
10	7	2	19
6	3	0	12
7	6	1	17
9	5	2	12
8	3	1	11
7	4	0	4

3.

℔	℥	ʒ	℈	gr.
12	11	6	1	15
4	4	1	0	12
91	10	7	2	16
4	8	1	2	19
9	6	3	0	7
3	4	5	2	9

5. CLOTH MEASURE.

Table.

2½ Inches,	*in.*	MAKE ONE	Nail, marked,	*n.* or *na.*
4 Nails, or 9 inches,			Quarter of a Yard,	*qr.*
4 Quarters, or 36 Inches,			Yard,	*Yd.*
3 Quarters, or 27 Inches,			Ell Flemish,	*E. Fl.*
5 Quarters, or 45 Inches,			Ell English,	*E. E.*
6 Quarters, or 54 Inches,			Ell French,	*E. Fr.*
4 Quarters, 1 In. & $\frac{1}{5}$, or 37 Inches and one 5th,			Ell Scotch,	*E. Sc.*
3 Quarters and two-thirds,			Spanish Var.	

---

95. *What is the use of Apothecaries' Weight?*—— 96. *What are its denominations?* ——97. *What are the varieties in Cloth Measure?*

EXAMPLES.

1.

Yd.	qr.	n.
76	2	3
3	3	1
42	3	3
57	2	2

2.

E.E.	qr.	n.
91	3	2
49	4	3
6	2	3
84	4	1

3.

E.Fl.	qr.	n.
75	2	1
7	1	3
84	0	2
76	2	3

Note.—The Scotch allow one English yard in every score yards.—All Scotch and Irish linens are bought by the English or American yard, which are the same, and all Dutch linens by the Ell Flemish; but are all sold in America by the American yard.

6. LONG MEASURE.

☞ The use of Long Measure is to measure the distances of places, or any thing, where length is considered without regard to breadth.

Table.

3	Barley-corns,	*bar.*	MAKE ONE	Inch, marked,	*in.*
12	Inches,			Foot,	*ft.*
3	Feet,			Yard,	*yd.*
5½	Yards, or 16½ feet,			Rod, Perch, or Pole,	*pol.*
40	Rods, Perches or Poles, or 132 Paces,			Furlong,	*fur.*
8	Furlongs,			Mile,	*mile.*
60	Geographical, or 69½ statute miles,			Degree,	*deg. or* °
360	Degrees,			Great Circle of the Earth.	

98. *What is long Measure?*——99. *Repeat the table?*

EXAMPLES.

1.

Pol.	ft.	in.
12	11	10
9	10	9
8	12	11
7	15	1

2.

Mil.	fur.	Pol.
9	7	36
7	3	19
4	1	24
6	5	12

3.

Deg.	mil.	fur.	pol.	ft.	in.	bar.
759	56	6	29	15	10	2
217	39	1	36	11	6	1
497	63	7	24	9	8	1
562	17	0	11	13	00	0

Note.—Distances are also measured with a chain, 4 rods long, containing 100 links.

$\frac{92}{100}$ inches = 1 link.
25 links = 1 pole.
100 links = 1 chain.
10 chains = 1 furlong.
8 furlongs = 1 mile.
4 inches = 1 hand.*
6 points = 1 line.

12 lines = 1 inch.
12 inches = 1 foot.
6 feet = 1 fathom.†
3 miles = 1 league.
5 feet = 1 geometrical pace.
66 feet = 1 Gunter's chain.

7. TIME.

Table.

60 Seconds	MAKE ONE	Minute, marked	*s. m.*
60 Minutes		Hour,	*h.*
24 Hours		Day,	*d.*
7 Days		Week,	*w.*
4 Weeks		Month,	*mo.*
13 Months 1 day & 6 hours, or 365 days & 6 hours,		Julian Year,	*Y.*

---

* Used in measuring the height of horses.

† Or French toise, used in measuring the depth of water and cordage, and is equal to 6 feet 4 inches English measure.

100. *In what other way are distances measured?*——101. *What is a chain?*——102. *What a furlong?*——103. *A fathom?*——104. *What is a league?*——105. *What is a geometrical pace?*——106. *What are the divisions of time?*

EXAMPLES.

1.

W.	d.	h.	m.	s.
3	6	22	57	42
1	5	19	31	28
2	3	17	9	15
3	0	9	17	58

2.

Y.	mo.	d.
19	10	19
7	9	27
4	8	16
1	11	14

3.

Y.	mo.	w.	d.	h.	m.	s.
57	11	3	6	23	29	55
4	8	1	1	19	45	38
29	9	2	3	17	18	19
46	10	2	5	11	50	13

NOTE.—The civil Solar year of 365 days being short of the true, by 5*h.* 48*m.* 57*s.* occasioned the beginning of the year to run forward through the season nearly one day in four years; on this account, Julius Cæsar ordained that one day should be added to February every fourth year, by causing the 24th day to be reckoned twice; and because this 24th day was the sixth, (sextilis) before the kalends of March, there were in this year two of these sextiles, which gave the name of Bissextile to this year, which being thus corrected, was, from thence, called the Julian year.

8. MOTION, OR CIRCULAR MEASURE.

☞ This measure is used in reckoning latitude and longitude; also in computing the revolution of the earth and other planets round the sun.

Table.

60 Seconds (″)	MAKE ONE	Minute,	′
60 Minutes		Degree,	°
30 Degrees		Sign,	*s.*
12 Signs, or 360 degrees,		Circle of the Zodiack.*	

---

* The Zodiack is the great circle of the Sphere, containing the 12 signs, through which the Sun appears to pass in a year.

107. *What is the origin of the Julian Year?*——108. *What is a degree?*——109. *What is the use of the table of Motion?*

EXAMPLES.

1.

°	′	″
17	55	48
1	37	51
29	19	45
19	19	37

2.

sig.	°	′	″
10	5	37	42
102	7	25	72
14	8	26	11
72	4	32	17
157	6	0	47

3.

sig.	°	′	″
104	7	26	16
648	5	27	4
293	6	17	13
136	3	38	24
498	5	42	19

9. LAND OR SQUARE MEASURE.

☞ By this are measured things that have both length and breadth.

Table.

		make one	
144	Inches		Square Foot.
9	Feet		Square Yard.
30¼ 272¼	Yards, or Feet		Square Pole.
40	Poles		Square Rood.
4	Roods, or 160 Rods, or 4840 Yards,		Square Acre.
640	Acres		Square Mile

EXAMPLES.

1.

Pole.	feet.	inch.
35	179	137
19	248	119
12	96	75
13	110	122

2.

Yard.	feet.	inch.
28	7	119
9	3	75
29	6	120
4	8	62

3.

Acre.	rood.	pole.	feet.	inch.
356	3	37	245	128
29	1	20	93	25
416	3	31	128	119
37	1	19	218	20

---

110. *What is a pole in Square Measure?*——111. *What a foot?*——112. *How many yards in an acre?*

10. SOLID MEASURE.

☞ This Measure is used to measure things that have length, breadth and thickness.

Table.

1728	Inches	make one	Foot.
27	Feet		Yard.
40	Feet of round timber, or		Ton or Load.
50	Feet of hewn timber,		
128	Solid feet, i. e. 8 in length, 4 in breadth, and 4 in height,		Cord of Wood.

EXAMPLES.

1.			2.			3.		
Ton.	ft.	in.	Yard	ft.	in.	Cord.	ft.	in.
29	46	1229	75	22	1412	37	119	1015
12	19	64	9	26	195	9	110	159
18	11	917	3	19	1091	48	127	1017
19	8	1002	28	15	1722	8	125	956

NOTE.—16 solid feet are called a foot in measuring wood, and 8 feet a cord. Also, 24¾ solid feet are called a perch in measuring stone.

11. WINE MEASURE.

☞ All brandies, spirits, perry, cider, mead, vinegar, honey and oil, are measured by wine measure.

Table.

				marked,
4	gills, (gi.)	MAKE ONE	pint,	*pt.*
2	pints,		quart,	*qt.*
4	quarts,		gallon,	*gal.*
31½	gallons,		barrel,	*bar.*
63	gallons,		hogshead,	*hhd.*
2	hogsheads,		pipe, or butt,	*P.* or *B.*
2	pipes, or 4 hogsheads,		ton,	*T.*

---

113. *What is a foot in Solid Measure?*—114. *What is a cord of wood?*—115. *How many feet of timber in a ton?*—116. *What is a perch?*—117. *What are the denominations of Wine Measure?*

EXAMPLES.

1.

Bbl.	gal.	qt.	pt.
37	29	3	1
9	17	2	1
4	28	0	0
32	19	1	1

2.

Hhd.	gal.	qt.	pt.
51	53	1	1
27	39	3	0
9	18	0	1
0	9	2	1

3.

Ton.	hhd.	gal.	qt.
37	2	37	2
19	1	59	1
28	2	0	0
19	0	47	1

Note.—The wine gallon contains 23 1 cubick inches.

12. ALE OR BEER MEASURE.

Table.

2 pints,	*pt.*	MAKE ONE	quart, marked,	*qt.*
4 quarts,			gallon,	*gal.*
8 gallons,			firkin of Ale in London.	*A. fir.*
8½ gallons,			firkin of Ale or Beer,	*fir.*
9 gallons,			firkin of Beer in Lond.	*B. fir.*
2 firkins,			kilderkin,	*kil.*
2 kilderkins,			barrel,	*bar.*
1½ barrel, or 45 gallons,			hogshead of Beer.	*hhd.*
2 barrels,			puncheon,	*pun.*
3 barrels, or 2 Hhds.			butt,	*butt.*

EXAMPLES.

1.

A.B.	fir.	gal.
49	3	7
26	2	3
9	0	4
17	3	0

2.

B.B.	fir.	gal.
29	1	8
19	3	5
16	0	3
9	1	8

3.

Hhd.	gal.	qt.
379	53	3
79	0	2
121	57	2
467	19	1

Note.—The beer gallon contains 282 cubick inches.

---

118. *How many cubick inches are there in a gallon of wine; and how many in a gallon of beer?*——119. *What are the divisions in Ale or Beer Measure?*

13. DRY MEASURE.

☞ This measure is applied to all Dry Goods, such as Corn, Seeds, Fruit, Roots, Salt, Sand, Coals, &c.

Table.

2 pints,	*pt.*	MAKE ONE	quart, marked,	*qt.*
2 quarts,			pottle,	*pot.*
2 pottles,			gallon,	*gal.*
2 gallons,			peck,	*pk.*
4 pecks,			bushel,	*bu.*
2 bushels,			strike,	*str.*
2 strikes,			coom,	*co.*
2 cooms,			quarter,	*qr.*
4 quarters,			chaldron,	*ch.*
4½ quarters,			chaldron in London.	
5 quarters,			wey,	*wey.*
2 weys,			last,	*last.*

EXAMPLES.

1.

Qr.	bu.	pk.	qt.
64	7	3	7
9	4	1	5
19	6	2	1
4	0	2	0

2.

Bu.	pk.	qt.	pt.
37	2	5	1
19	3	7	0
16	2	0	1
5	1	6	1

3.

Chal.	bu.	pk.	qt.
37	27	3	5
6	29	1	7
15	30	0	0
4	11	3	0

NOTE.—A Winchester bushel is 18½ inches in diameter, 8 inches deep, and contains 2,150⅖ cubick inches.

☞ A barrel of Mackerel, and other barrelled fish, by law in New-Hampshire, is to contain not less than 29, nor more than 30 gallons. In Massachusetts, it is 30 gallons. In Connecticut and New-York, the Shad and Salmon barrel, must contain 200 lbs.

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120. *What is the use of Dry Measure?*——121. *Repeat the table.*

COMPOUND SUBTRACTION.

Compound Subtraction teaches to find the difference between any two sums of divers denominations.

RULE.

Place the numbers under each other, which are of the same denomination, the less being below the greater; begin with the least denomination, and if it exceed the figure over it, borrow as many units as make one of the next greater; subtract it therefrom; and to the difference add the upper figure, remembering always, to add one to the next superior denomination, for that which you borrowed.*

Proof.—The same as in simple subtraction.

1. FEDERAL MONEY.

EXAMPLES.

	1.			2.				3.		
	D.	c.	m.	E.	D.	c.	m.	D.	c.	m.
From	39	15	5	21	8	1	2	100	00	0
Take	28	17	2	10	7	5	0	48	87	5

	D.	c.	m.
Lent	7159	12	8
Received at several times.	945	37	5
	3112	15	7
	2000	00	0
	1092	92	0
Rec'd in all			
Remains due			

First add together the several sums received, then subtract their amount from the sum lent.

---

* The reason of this rule will readily appear, from what was said in subtraction of simple numbers; for the borrowing depends upon the same principle, and is only different as the numbers to be subtracted are of different denominations.

122. *What is Compound Subtraction?*——123. *What is the rule?*——124. *And the method of proof?*——125. *What is the difference between simple and compound subtraction?*

2. ENGLISH MONEY.

1.

	£.	s.	d.	qrs.
Borrowed	349	15	6	1
Paid	195	11	8	1

2.

	£.	s.	d.	qrs.
Lent	291	9	8	1
Received	107	16	4	2

3. TROY WEIGHT.

1.

℔.	oz.	pwt.	gr.
749	5	13	16
91	9	19	13

2.

℔.	oz.	pwt.	gr.
189	8	12	10
143	4	11	19

3.

lb.	oz.	pwt.	gr.
543	3	9	13
179	1	15	18

4. AVOIRDUPOIS WEIGHT.

1.

lb.	oz.	dr.
7	9	12
3	12	9

2.

Cwt.	qrs.	lb.
8	2	13
4	1	15

3.

T.	cwt.	qrs.	lb.	oz.	dr.
9	11	3	17	5	12
3	12	1	19	10	9

5. APOTHECARIES' WEIGHT.

1.

lb.	℥.	ʒ.	℈.	gr.
71	9	3	1	13
37	8	4	1	16

2.

lb.	℥.	ʒ.	℈.	gr.
65	10	6	2	1
31	8	4	2	0

3.

lb.	℥.	ʒ.	℈.	gr.
84	1	1	1	1
65	9	3	1	17

6. CLOTH MEASURE.

1.

Yd.	qr.	n.
35	1	2
19	1	3

2.

E.E.	qr.	n.
467	3	1
291	3	2

3.

E.Fl.	qr.	n.
765	1	3
149	2	1

4.

E.Fr.	qr.	n.
549	4	2
197	4	3

7. LONG MEASURE.

1.

Yd.	ft.	in.
28	2	10
17	2	11

2.

Mil.	fur.	pol.
76	3	11
27	3	21

3.

Deg.	mil.	fur.	pol.	yd.	ft.	in.	bar.
38	41	3	29	2	1	7	2
19	35	5	31	3	1	9	1

8. TIME.

1.

W.	d.	h.	m.	s.
6	17	13	27	19
1	21	16	41	35

2.

Y.	mo.	d.
7	3	13
4	4	19

3.

Y.	mo.	w.	d.	h.	m.	s.
48	9	2	5	19	27	31
19	9	3	4	20	19	49

9. MOTION.

1.

°	′	″
79	21	31
41	41	52

2.

s.	°	′	″
6	11	17	48
3	8	39	29

3.

s.	°	′	″
4	19	41	22
1	22	19	45

10. LAND OR SQUARE MEASURE.

1.

Acre.	rood.	pole.
29	1	10
24	1	25

2.

Acre.	rood.	pole.
29	3	19
17	1	36

3.

Acre.	rood.	pole.	feet.	inch.
56	2	16	27	110
29	0	21	210	129

11. SOLID MEASURE.

1.

Ton.	ft.	in.
49	19	1100
38	36	1296

2.

Yds.	ft.	in.
79	11	917
17	25	1095

3.

Cord.	ft.	in.
349	97	1250
192	127	1349

12. WINE MEASURE.

1.

Hhd.	gal.	qt.	pt.
79	21	2	1
38	61	3	1

2.

Tier.	gal.	qt.
19	17	1
12	29	2

3.

Tun.	hhd.	gal.
532	1	19
197	1	47

13. ALE AND BEER MEASURE.

1.

A.B.	fir.	gal.	qt.
39	1	2	1
24	3	6	2

2.

B.B.	fir.	gal.	qt.	pt.
21	3	5	2	0
14	1	7	2	1

3.

Hhd.	gal.	qt.
769	17	1
391	42	3

14. DRY MEASURE.

1.

Qr.	bu.	pk.	qt.
56	2	2	1
39	3	1	2

2.

Bu.	pk.	qt.	pt.
91	1	3	2
29	2	1	1

3.

Chal.	bu.	pk.	qt.
39	12	2	1
24	25	3	2

* * *

PROBLEMS,

RESULTING FROM A COMPARISON OF THE PRECEDING RULES.

[☞ By a due consideration and application of these Problems, many questions may be solved in a short and easy manner, although some of them are generally supposed to belong to higher rules.]

Problem 1. Having the sum of two numbers, and one of them given, to find the other.

Rule.—Subtract the given number from the given sum, and the remainder will be the sum required.

Let 288 be the sum of two numbers, one of which is 115, the other is required.

From 288 the sum,
Take 115 the given number:

Remains 173 the other.

Prob. 2. Having the greater of two numbers, and the difference between that and the less given, to find the less.

Rule.—Subtract the one from the other.

Let the greater number be 325, and the difference between that and the other 198: What is the other?

From 325 the greater,
Take 198 the difference.

Remainder 127 the less.

PROB. 3. Having the lesser of two numbers given, and the difference between that and a greater, to find the greater.

Rule.—Add them together.

Given { 127 the less number,
Given { 198 the difference.

Sum 325 the greater number required.

PROB. 4. Having the sum and difference of two numbers given, to find those numbers.

Rule.—To half the sum add half the difference, and the sum is the greater; and from half the sum take half, the difference, and the remainder is the less.—Or, from the sum take the difference, and half the remainder is the least; to the least add the given difference, and the sum is the greatest.

What are those two numbers, whose sum is 48, and difference 14?

2)48 2)14

Half sum=24 Half difference=7

24+7=31 the greater; and 24—7=17 the least,

Or, 48—14÷2=17; and 17+14=31.

PROB. 5. Having the sum of two numbers and the difference of their squares* given, to find those numbers.

Rule.—Divide the difference of their squares by the sum of the numbers, and the quotient will be their difference: You will then have their sum and difference, to find the numbers by Problem 4.

What two numbers are those, whose sum is 32, and the difference of whose squares is 256?

Half sum 16
Half diff. 4

32)256(8 difference.
256

Greater 20

Less 12

PROB. 6. Having the difference of two numbers, and the difference of their squares given, to find those numbers.

---

* The square of a number is the product of it multiplied into itself.

Rule.—Divide the difference of their squares by the difference of the numbers, and the quotient will be their sum; then proceed by Problem 4.

What are those two numbers, whose difference is 20, and the difference of whose squares is 2000?

20)2000(100 sum. 50+10=60, the greater; and
50—10=40 the less.

Prob. 7. Having the product of two numbers, and one of them given, to find the other.

Rule.—Divide the product by the given number, and the quotient will be the number required.

Let the product of two numbers be 288, and one of them 8; we demand the other.

8)288
———
Ans. 36

Prob. 8. Having the dividend and quotient, to find the divisor.

Rule.—Divide the dividend by the quotient.

☞ Hence we get another method of proving division.

Given { 288 the dividend,
36 the quotient.
Required the divisor.

36)288(8 divisor.
288
——

Prob. 9. Having the divisor and the quotient given, to find the dividend.

Rule.—Multiply them together.

Given { 8 the divisor,
36 the quotient.
Required the dividend.

36
8
——
288 the dividend.

Remark.—The scholar, having now surveyed the ground-work of his studies, begins to see their application to the common concerns of life;—and it is important, while proceeding in the higher rules, that his memory be strengthened by repeated examinations in the previous studies. The instructor is advised, therefore, to state questions of his own, promiscuously under the several rules, that the good scholar may have an opportunity of proving to his teacher and friends, by prompt and ready answers to difficult questions, that he thoroughly understands the subject before him.—This hint, improved *now*, may be of essential service hereafter.

REDUCTION.

REDUCTION is the method of changing numbers of one denomination into others of different denominations, without altering their value.

It is of *two* sorts, viz. Descending and Ascending.

REDUCTION DESCENDING

Teaches to change numbers from a higher denomination to a lower; and is performed by *multiplication.*

RULE.

Multiply the highest denomination given, by so many of the next less, as make one of that greater, and thus continue, till you have brought it down as low as your question requires.*

PROOF.

Change the order of the question, and divide your last product by the last multiplier, and so on.

NOTE.—From this rule and Case II. of Simple Multiplication, it appears that Federal Money is reduced from higher to lower denominations, by annexing as many ciphers as there are places from the denomination given to that required; or, if the given sum be of different denominations, by annexing the several figures of all the denominations in their order, and continuing with ciphers, if necessary, to the denomination required; or, what amounts to the same thing, by reading the whole number from the left to the required denomination, as one number in the required denomination.

EXAMPLES.

1. In 3 eagles 2 dollars, how many mills? Ans. 32000 m.

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* The reason of this rule is obvious; for pounds are brought into shillings by multiplying them by 20; shillings into pence by multiplying them by 12; and pence into farthings by multiplying them by 4; and this will be true in the reduction of numbers consisting of any denomination whatever. The reason of the rule in Reduction Ascending, is equally evident; for farthings are brought into pence by dividing them by 4; pence into shillings by dividing them by 12; and shillings into pounds by dividing them by 20.

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126. *What is Reduction?*—127. *Of how many parts does it consist?*—128. *What is Reduction Descending?*—129. *What is the rule for changing any number from a greater to a less denomination?*—130. *What is the reason of this rule?*

2. In 91 dollars 75 cents, how many cents?
Ans. 9175c.

3. In 50 Eagles, how many dollars? Ans. $500.

4. In 44 dollars 1 cent 4 mills, how many mills?

5. In 9 dollars 37 cents 7 mills, how many mills?

6. How many cents in 39 dollars 5 cents?

7. In £27 15s. 9d. 2qrs. how many farthings?

£. *s.* *d.* *qrs.*
27 15 9 2
Multiplied by 20=shillings in a pound.
555=shillings.
Multiplied by 12=pence in a shilling.
6669=pence.
Multiplied by 4=farthings in a penny.
Ans.=26678

☞ In multiplying by 20, we add in the 15s.; by 12, the 9d. and by 4, the 2qrs. which must always be done in like cases.

Proof.
4)26678
12)6669 2 qrs.
2|0)55|5 9d.
Ans. £27 15 9 2

☞ This is in fact, in the present case, simply changing the order of the question, thus: In 26678 farthings, how many pounds?

8. In £36 12s. 10d. 1qr. how many farthings? *Ans.* 35177.

9. In £719 9s. 11d. how many half pence? *Ans.* 345358.

10. In 37 pistoles, at 22s. how many shillings, pence and farthings?
Ans. 814 shill.
9768 pence.
39072 farthings.

11. In 53 moidores, at 36s., how many shillings, pence and farthings?
Ans. 1908s.
22896d.
91584qrs.

12. Reduce 47 guineas, (at 28s. each,) and one fourth of a guinea into shillings, six-pences, groats, (4d.) three-pences, two-pences, pences, and farthings.

Ans. 1323 shillings, 2646 six-pences, 3969 groats, 5292 three-pences, 7938 two-pences, 15876d. and 63504qrs.

REDUCTION ASCENDING,

Teaches how to change numbers from a lower to a higher denomination ; and is performed by *division.*

Rule.

Divide the lowest denomination given, by so many of that name, as make one of the next higher, and thus continue, till you have brought it into that denomination, which your question requires.

Note.—From this rule and the note under Case II. of Simple Division, it appears, that FEDERAL MONEY is reduced from lower to higher denominations by cutting off as many places as the given denomination stands to the right of that required; the figures cut off belonging to their respective denominations.

EXAMPLES.

1. How many eagles in 32000 mills ? Ans. 3E. 2D.

2. In 9175 cents, how many dollars ? Ans. 91D. 75C.

3. In 500 dollars, how many eagles ? Ans. 50

4. In 4414 mills, how many dimes ?

5. In 9317 mills, how many dollars ?

6. In 39050 mills, how many dollars ?

7. In 547325 farthings, how many pence, shillings, and pounds ?

---

131. *How do you change numbers of a lower denomination into a higher ; or, in other words, how will you reduce farthings to pounds, or mills to eagles ?*

Farthings in a penny = 4)547325

Pence in a shilling = 12)136831 1qr.

Shillings in a pound = 2|0)1140|2 7d.

£570 2s. 7d. 1qr.

Ans. 136831d. : 11402s. ; £570.

Note.—The remainder is always of the same denomination with the dividend.

8. Bring 35177 farthings into pounds.

9. Bring 365358 half pence into pence, shillings and pounds.

10. In 39072 farthings, how many pistoles, at 22s. ?

11. In 63504 farthings, how many pence, two-pences, three-pences, groats, six-pences, shillings, and guineas ?

Note.—These questions may serve as proofs to some of those in Reduction Descending.

REDUCTION ASCENDING AND DESCENDING.

1. MONEY.

1. In £97, how many pence, and English or French Crowns, at 6s. 8d. ? *Ans.* 23280*d.* & 291 *Crowns.*

2. In 735 French Crowns, how many shillings and French guineas, at 26s. 8d. ? *Ans.* 4900*s.*, & 183 *guin.* 24*s.*

Note.—*When it is required to know how many sorts of coin, of different values, and of equal numbers, are contained in any number of another kind ;* reduce the several sorts of coin into the lowest denomination mentioned, and add them together for a divisor ; then reduce the money given into the same denomination for a dividend, and the quotient arising from the division will be the number required.

☞ Observe the same direction in weights and measures.

---

132. *I wish to ascertain how many moidores, guineas, pistoles, dollars, shillings and six-pences, each of the like number, are contained in 275 half joes : how shall I proceed ?*

3. In 275 half Johannes, how many moidores, guineas, pistoles, dollars, shillings, and six-pences, of each the like number ?

A moid. is 36s.=72 six-pences.
A guin. is 28s.=56 do.
A pistole is 22s.=44 do.
A dollar is 6s. =12 do.
One shilling is =2 do.
1 do.
———
Divisor=187 six-pences.

275 half joes.
48s. in a ½ johan.
———
2300
1100
———
13200 shillings.
2 six-p. in a shill.
———
Dividend=26400 six-pences.

187)26400(141 of each, and 33 six pences, or 16s. 6d. over, the answer.

4. A gentleman distributed £37 10s. between 4 persons in the following manner, viz. that as often as the first had 20s. the second should have 15s. the third 10s. and the fourth 5s. What did each person receive ?

Ans. The first man, £15.
The second, £11 5s.
The third, £7 10s.
The fourth, £3 15s.

2. TROY WEIGHT.

1. How many grains in a silver bowl, that weighs 3 lbs. 10 oz. 12 pwt. ?

℔ oz. pwt.
3 10 12
12 ounces in a pound.
——
46 ounces.
20 pennyweights in an ounce.
——
932 pennyweights.
24 grains in one pennyweight.
——
3728
1864
——
Proof. 24)22368 grains, *Ans.*
——
2|0)93|2
——
12)46 |12 pwt.
——
3℔ 10 oz.

2. In 13 ingots of gold, each weighing 9 oz. 5 pwt. how many grains? *Ans.* 57720 grains.

3. In 97397 grains, how many pounds?
Ans. 16 lbs. 10 oz. 18 pwt. 5 gr.

4. How many rings, each weighing 5 pwt. 7 gr. may be made of 3 lbs. 5 oz. 16 pwt. 2 gr. of gold? *Ans.* 158.

3. AVOIRDUPOIS WEIGHT.

```
       Cwt. qrs. ℔  oz.
1.  In 91    3   17  14 how many ounces?
          4
        ———
        367 quarters.                    Proof.
         28                            16)164702
        ———                               ———————
       2943                            28)10293  14 oz.
       735                                ——————
      ———                                  4)367  17 ℔.
      10293 pounds.                         ————
         16                           Cwt. 91  3 qrs.
      ———
      61762
     10293
     ——————
     164702 ounces.
```

2. In 12 tons, 15 cwt. 1 qr. 19 lbs. 6 oz. 12 dr. how many drams? *Ans.* 7323500 drams.

3. In 44800 pounds, how many drams and tons?
Ans. 11468800 dr. and 20 tons.

4. In 47 lbs. 9 oz. 13 pwt. 17 grs. Troy, how many pounds Avoirdupois ?

lbs.	oz.	pwt.	gr.
47	9	13	17

12
573
20
11473
24
45899
22947
7|000)275|369(39 lbs.
21
65
63
2369
16
14214
2369
7|000)37|904(5 oz.
35
2904
16
17424
2904
7|000)46|464(6$\frac{4464}{7000}$ drams.
42
4464

4. LONG MEASURE.

1. How many barley-corns will reach from Newburyport to Boston, it being 43 miles?

Miles.	Proof.
43	3)8173440
8	12)2724480
344 furlongs.	3)227040
40	11)75680
13760 rods.	6880
5½	2
68800	4|0)1376|0
6880	8)344
72680 yards.	43
3	
227040 feet.	
12	
2724480 inches.	
3	
8173440 *Ans.*	

☞ Here we divide by 11, and multiply the quotient by 2, because twice 5½ is 11; or we might have first multiplied by 2, and then have divided the product by 11.

2. How many barley corns will reach round the globe, it being 360 degrees?

Ans. 4755801600.

3. How many inches from Newburyport to London, it being 2700 miles?

Ans. 171072000.

4. How many yards, feet and inches from Concord, N. H. to the City of Washington, it being 498 miles?

5. TIME.

1. In 20 years, how many seconds?

d. h.
365 6 in a year.
24

1456
730

8766 hours in one year.
20

175320 hours in 20 years.
60

10519200 minutes in do.
60

631152000 seconds in do.

Proof.
6|0)63115200|0

6|0)1051920|0

2|0)17532|0

4 × 6)8766

4)1461

365d. 6h.

2. Suppose your age to be 15y. 19d. 11h. 37m. 45s., how many seconds are there in it, allowing 365 days 6 hours to the year?
Ans. 475047465.

3. The war of the Revolution commenced April 19, 1775, and Independence was declared July 4, 1776: how many seconds between?

6. LAND OR SQUARE MEASURE.

1. In 29 acres, 3 roods, 19 poles, how many roods and perches?

29 3 19
4

119 roods.
40

Ans. 4779 perches.

4|0)477|9

4)119 19p.

29a. 3r.

2. In 1997 poles, how many acres ?

Ans. 12a. 1r. 37p.

3. How many square feet, square yards, and square poles, in a square mile ?

Ans. 27878400 feet.
3097600 yards.
102400 poles.

7. SOLID MEASURE.

1. In fifteen tons of hewn timber, how many solid inches ?

15 tons.
50
———
750 feet.
1728
———
6000
1500
5250
750
———
1296000 inches.....*Ans.*

PROOF.
5|0
1728)1296000(75|0
12096
———
8640
8640
———
15 tons.

2. In 25 cords of wood, how many inches ?

Ans. 5529600.

☞ GRINDSTONES ARE USUALLY SOLD BY THE SOLID FOOT.

The contents may be found by the following

Rule.—Multiply the sum of the whole diameter by the half diameter, and this product by the thickness, and you have the product in cubick inches.

---

133. *How are grindstones measured and sold ?*

3. What is the solid content of a grindstone 32 inches in diameter, and three inches thick?

32 diameter.	1728)2304(1 foot.
16 half diameter.	1728
48	576
16	3
768	)1728(1 third.
3	1728
2304 solid inches.	Ans. 1⅓ feet.

8. WINE MEASURE.

1. In 9 hhd. 15 gal. 3 qts. of wine, how many quarts?

9 15 3	PROOF.
63	4)2331
32	63)582 3 qts.
55	9 hhd. 15 gal.
582	
4	
2331 quarts....Ans.	

2. In twelve pipes of wine, how many pints?

Ans. 12096.

Remark.—It is thought unnecessary to give examples of reduction under all the tables of Weights, Measures, &c.; as the attentive scholar will readily understand the correct process, from the exercises he has already had. The learner may, however, find it useful to turn to those tables under which no example is here given—state questions of his own accord—and, having found their true answers, write them down with the other examples in his Ciphering-Book.

FRACTIONS.*

Fractions are parts of an unit, or whole number. When a *whole* is expressed by figures, the number is called an *integer* :—but when a part, or some parts, of a thing are denoted by figures, as, $\frac{1}{4}$ *one fourth*, $\frac{2}{3}$ *two thirds*, $\frac{3}{10}$ *three tenths*, &c. of a thing, these figures are called *fractions*.

Fractions are divided into two kinds, **Vulgar** and **Decimal**.

VULGAR FRACTIONS.

A **Vulgar Fraction** is that which can have any denominator ; and is expressed by two numbers written one above the other, thus—$\frac{5}{8}$, with a line between.

The figure above the line is called the *numerator*,
The figure below the line is called the *denominator*, $\Big\{\frac{5}{8}$

The denominator (which is the divisor in division) shows how many parts the integer is divided into ;—and the numerator (which is the remainder after division) shows how many of those parts are meant by the fraction.

Fractions are either proper, improper, single, compound, or mixed. Any whole number may be made an improper fraction, by drawing a line under it, and putting unity, or 1, for a denominator ; as, 9 may be expressed, fractionwise, thus—$\frac{9}{1}$, and 12 thus, $\frac{12}{1}$, &c.

1. A *single*, or *simple* fraction, is a fraction expressed in a simple form ; as, $\frac{1}{2}$, $\frac{5}{9}$, $\frac{7}{10}$, &c.

2. A *compound* fraction is a fraction expressed in a compound form, being a fraction of a fraction, or, two or more fractions connected together ; as, $\frac{1}{2}$ of $\frac{3}{4}$, $\frac{2}{7}$ of $\frac{5}{11}$ of $\frac{19}{20}$; which are read thus—one half of three fourths, two sevenths of five elevenths of nineteen twentieths, &c.

---

* The term, FRACTION, signifies a broken part or parts of any thing or number ; and these parts can be represented by figures, as well as whole things or numbers. It was shewn on page 29, that fractions arise from the operations of *division ;* and hence we may see the necessity of understanding something of the arithmetick of Vulgar Fractions, even though it be in some respects " a tedious and intricate rule."

134. *What are fractions ?*——135. *What is a Vulgar Fraction ?*——136. *How is it written ?*——137. *What is the figure above the line called ?*——138 *What is the one below the line called ?*——139. *What is the meaning, or the use, of these terms ?*——140. *Are there different kinds of Vulgar Fractions ?*——141. *How can a whole number be made an improper fraction ?*——142. *What is a simple fraction ?*——143. *What is a compound fraction ?*

3. A *proper* fraction is a fraction, whose numerator is less than its denominator; as, $\frac{1}{4}$, $\frac{2}{3}$, $\frac{3}{8}$, &c.

4. An *improper* fraction is a fraction, whose numerator is larger than its denominator; as, $\frac{5}{2}$, $\frac{7}{3}$, $\frac{9}{5}$, &c.

5. A *mixed number* is composed of a whole number and a fraction; as, $7\frac{3}{5}$, $35\frac{4}{13}$, &c., that is, seven and three fifths, &c.

6. The *common measure* of two or more numbers, is that number, which will divide each of them without a remainder: Thus, 5 is the common measure (or divisor) of 10, 20, and 30; and the *greatest* number which will do this is called the *greatest common measure.*

7. A number, which can be measured by two, or more, numbers, is called their *common multiple*: And, if it be the least number which can be so measured, it is called the *least common multiple;* thus, 40, 60, 80, 100, are multiples of 4 and 5; but their least common multiple is 20.

8. A *prime number* is that, which can only be measured (that is, divided) by itself, or an unit; as, 5, &c.

9. A *perfect number* is equal to the sum of all its aliquot parts.*

PROBLEM I.

To find the greatest common measure of two or more numbers.

Rule 1. If there be two numbers only, divide the greater by the less, and this divisor by the remainder, and so on, always dividing the last divisor by the last remainder, till nothing remain; then will the last divisor be the greatest common measure required.

2. When there are more than two numbers, find the greatest common measure of two of them, as before; then of that com-

---

* The *aliquot parts* are the parts of a number, that will measure it without any remainder; as, 3 is an aliquot part of 12. And 2, 3, and 6, are aliquot parts of 6. And 2, 4, 7, 14, and 28, are aliquot parts of 28. Now, if 6 be divided by 2, 3, and 6, the respective quotients will be 3, 2, and 1, whose sum is equal to 6, the original number. And if 28 be divided by 2, 4, 7, 14, and 28, the several quotients will be 14, 7, 4, 2, and 1, whose sum is equal to 28, the given number. 6 and 28 are therefore perfect numbers, according to the definition. The sum of all the aliquot parts of any number, which is not a perfect one, will be either less or greater than the given number, and never precisely equal to it; as may be proved by a similar operation.

---

144. *What is a proper fraction?*——145. *What is an improper fraction?*——146. *What is a mixed number?*——147. *What is a common measure of two or more numbers?*——148. *What is a common multiple?*——149. *What is the difference between a prime and a perfect number?*——150. *How do you find the greatest common measure of two or more numbers?*

mon measure and one of the other numbers, and so on, through all the numbers, to the last; then will the greatest common measure, last found, be the answer.

3. If 1 happens to be the common measure, the given numbers are prime to each other, and found to be incommensurable, or in their lowest terms.*

EXAMPLES.

1. What is the greatest common measure of 36 and 96?

```
                         36)96(2
                            72
                           ----
                            24)36(1
                               24
                              ----
Greatest common measure = 12)24(2
                             24
                            ----
```

☞ It is evident, that 12 is the greatest number that will divide 36 and 96 without a remainder;—therefore, it is the greatest common measure, or answer.

2. What is the greatest common measure of 1224 and 1080?
Ans. 72.

PROBLEM II.

To find the least common multiple of two or more numbers.

Rule 1. Divide by any number that will divide two or more of the given numbers without a remainder, and set the quotients, together with the undivided numbers, in a line beneath.

2. Divide the second line as before, and so on, till there are no two numbers that can be divided; then the continued product of the divisors and quotients will give the multiple required.†

---

* The truth of this rule may be shown from the first example; for since 12 divides 24, it also divides $24+12$, or 36. Again, since 12 divides 24 and 36, it also divides $36\times2+24$, or 96.

† The reason of this rule may be shown from the first example, thus: It is evident, that $3\times5\times8\times10=1200$ may be divided by 3, 5, 8, and 10, without a remainder; but 10 is a multiple of 5; therefore, $3\times5\times8\times2=240$ is also divisible by 3, 5, 8, and 10.—Also, 8 is a multiple of 2; therefore, $3\times5\times4\times2=120$ is also divisible by 3, 5, 8, and 10; and is evidently the least number that can be so divided.

---

151. *What is the rule for finding the greatest common multiple of two or more numbers?*

EXAMPLES.

1. What is the least common multiple of 6, 10, 16, and 20?

*5)6 10 16 20
*2)6 2 16 4
*2)3 1 8 2
*3 1 *4 1

* * * * *
5×2×2×3×4=240 *Ans.*

☞ We survey the given numbers, and find that 5 will divide two of them, viz. 10 and 20, which we divide by 5, bringing into a line with the quotients, the numbers, which 5 will not measure: again, we view the numbers in the second line, and find 2 will measure them all, and we get 3, 1, 8, 2, in the third line, and find that 2 will measure 8 and 2, and in the fourth line get 3, 1, 4, 1 all prime; we then multiply the prime numbers and the divisors continually into each other, for the number sought, and find it to be 240.

2. What is the least number, which can be divided by the 9 digits separately, without a remainder? Ans. 2520.

REDUCTION OF VULGAR FRACTIONS,

Is the bringing them out of one form into another, in order to prepare them for the operations of Addition, Subtraction, &c.

CASE I.

*To abbreviate or reduce fractions to their lowest terms.**

---

* A fraction is in its lowest terms, when represented by the least numbers possible: thus $\frac{5}{10}$ when reduced to its lowest terms is $=\frac{1}{2}$.

152. *What is reduction of vulgar fractions?*——153. *When is a fraction in its lowest terms?*

RULE.

Divide the terms of the given fraction by any number, which will divide them without a remainder, and the quotients again in the same manner; and so on, till it appears there is no number greater than 1, which will divide them, and the fraction will be in its lowest terms.

Or, divide both the terms of the fraction by their greatest common measure, and the quotients will be the terms of the fraction required.

NOTE.—That dividing both the terms, that is, both numerator and denominator of the fractions, equally by any number, whatever, will give another fraction, equal to the former, is evident; and if those divisions be performed as often as can be done, or the common divisor be the greatest possible, the terms of the resulting fraction must be the least possible.

1. Any number ending with an even number or cypher is divisible by 2.

2. Any number ending with 5, or 0, is divisible by 5.

3. If the right hand figure of any number be 0, the whole is divisible by 10.

4. If the two right hand figures of any number be divisible by 4, the whole is divisible by 4.

5. If the sum of the digits constituting any number, be divisible by 3, or 9, the whole is divisible by 3 or 9.

6. All prime numbers, except 2 and 5, have 1, 3, 7, or 9, in the place of units, and consequently all other numbers are composite and capable of being divided.

EXAMPLES.

1. Reduce $\frac{288}{480}$ to its lowest terms.

$$8 \left\} \quad \frac{288}{480} = \overset{(4)}{\frac{36}{60}} = \overset{(3)}{\frac{9}{15}} = \frac{3}{5} \right.$$ the answer.

---

154. *What are the rules for reducing fractions to their lowest terms?*

or thus :

288)480(1
288
—
192)288(1
192
—
Common measure 96)192(2
192
—

Therefore 96 is the greatest common measure ;

and 96)$\frac{288}{480}=\frac{3}{5}$ the same as before.

2. Reduce $\frac{57}{456}$ to its lowest terms. Ans. $\frac{1}{8}$

3. Reduce $\frac{46}{184}$ to its lowest terms. Ans. $\frac{1}{4}$

4. Reduce $\frac{1428}{2856}$ to its lowest terms. Ans. $\frac{1}{2}$

CASE II.

To reduce a mixed number to its equivalent improper fraction.

Rule.*—Multiply the whole number by the denominator of the fraction, and add the numerator of the fraction to the product; under which subjoin the denominator, and it will form the fraction required.

---

[*] All fractions represent a division of a numerator by the denominator, and are taken altogether as proper and adequate expressions of the quotient. Thus the quotient of 3, divided by 4, is $\frac{3}{4}$.

155. *How do you reduce a mixed number to its equivalent improper fraction?*

EXAMPLES.

1. Reduce $36\frac{5}{8}$ to its equivalent improper fraction.

$$\begin{array}{r} 36 \\ \times 8+5 \\ \hline \text{Answer, } 293 \\ \hline 8 \end{array}$$

We multiply 36 by 8, and adding the numerator 5 to the product, as we multiply, the sum, 293, is the numerator of the fraction sought, and 8 the denominator; so that $\frac{293}{8}$ is the improper fraction, equal to $36\frac{5}{8}$.

$$\text{Or, } \frac{36\times8+5}{8}=\frac{293}{8} \text{ Answer as before.}$$

2. Reduce $127\frac{4}{17}$ to its equivalent improper fraction.
Ans. $\frac{2163}{17}$

3. Reduce $653\frac{3}{19}$ to its equivalent improper fraction.
Ans. $\frac{12410}{19}$

CASE III.

To reduce an improper fraction to its equivalent whole or mixed number.

Rule.—Divide the numerator by the denominator, the quotient will be the whole number, and the remainder, if any, written over the given denominator, will form the fractional part.

Note.—This rule is, evidently, the reverse of the rule in Case II., and has its reason in the nature of common division.

---

186. *What is the rule for reducing an improper fraction to its equivalent whole or mixed number?*

EXAMPLES.

1. Reduce $\frac{293}{8}$ to its equivalent whole or mixed number.

$8)293(36\frac{5}{8}$ Ans.
24
―
53
48
―
5

Or, $\frac{293}{8}=293\div8=36\frac{5}{8}$ as before.

2. Reduce $\frac{2163}{17}$ to its equivalent whole or mixed number.
Ans. $127\frac{4}{17}$.

3. Reduce $\frac{45}{5}$ to its equivalent whole number. Ans. 9.

CASE IV.

To reduce a compound fraction to an equivalent simple one.

RULE.—Multiply all the numerators continually together for a new numerator, and all the denominators, for a new denominator, and they will form the simple fraction required.

NOTE.—If any part of the compound fraction be a whole or mixed number, it must first be reduced to an improper fraction.

That a compound fraction may be represented by a single one, is evident, since a part of a part must be equal to some part of the whole. The truth of the rule for this reduction may be shewn as follows:

Let the compound fraction to be reduced be $\frac{2}{3}$ of $\frac{4}{7}$. Then $\frac{1}{3}$ of $\frac{4}{7}$ $=\frac{4}{7}\div3=\frac{4}{21}$, and consequently $\frac{2}{3}$ of $\frac{4}{7}$ $\times2=\frac{8}{21}$ the same as by the rule, and the like will be found true in all cases.

If the compound fraction consist of more numbers than two, the two first may be reduced to one, and that one and the third will be the same as a fraction of two numbers, and so on.

---

157. *How do you reduce a compound to a simple fraction?*

EXAMPLES.

1. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a simple fraction.

$$\frac{1\times2\times3\times4}{2\times3\times4\times5}=\frac{24}{120}=\frac{1}{5} \text{ the Ans.}$$

Or, by expunging the equal numerators and denominators, it will give $\frac{1}{5}$ as before.

2. Reduce $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{11}{12}$ to a simple fraction.

$$\frac{3\times4\times5\times11}{4\times5\times6\times12}=\frac{660}{1440}=\frac{11}{24} \textit{ Ans.}$$

Or, by expunging the equal numerators and denominators, it will be

$$\frac{3\times11}{6\times12}=\frac{33}{72}=\frac{11}{24} \text{ as before.}$$

3. Reduce $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{3}{4}$ of $12\frac{1}{2}$ to a simple fraction.

Ans. $\frac{75}{64}=1\frac{11}{64}$.

CASE V.

To reduce fractions of different denominators to equivalent fractions, having a common denominator.

Rule.—Multiply each numerator into all the denominators, except its own, for a new numerator, and all the denominators into each other continually, for a common denominator.

---

158. *What is the rule for reducing fractions of different denominators to equivalent fractions, having a common denominator?*

EXAMPLES.

1. Reduce $\frac{1}{4}$, $\frac{2}{5}$, and $\frac{5}{8}$ to equivalent fractions, having a common denominator.

$1\times5\times8=40$ the new numerator for $\frac{1}{4}$
$2\times4\times8=64$ ditto $\frac{2}{5}$
$5\times4\times5=100$ ditto $\frac{5}{8}$
$4\times5\times8=160$ the common denominator.

Therefore, the new equivalent fractions are $\frac{40}{160}$, $\frac{64}{160}$, and $\frac{100}{160}$, the answer.

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$, to fractions, having a common denominator.

Ans. $\frac{576}{1152}$, $\frac{768}{1152}$, $\frac{864}{1152}$, $\frac{960}{1152}$, $\frac{1008}{1152}$.

NOTE.—When there are whole numbers, mixed numbers, or compound fractions, given in the question, they must first be reduced to their simple forms.

CASE VI.

To reduce any given fractions to others, which shall have the least common denominator.

RULE.—Find the least common multiple (by problem II, page 72,) of all the denominators of the given fractions, and it will be the common denominator required ; then divide the common denominator by the denominator of each fraction, and multiply the quotient by the numerator for a new numerator ; the new numerators written over the common denominator will form the fractions required in their lowest terms.

---

159. *What is the rule for reducing any given fractions to others which shall have the the least common denominator ?*

NOTE.—The common denominator is a multiple of all the denominators, and consequently will divide by any of them ; therefore, proper parts may be taken for all the numerators as required by the rule.

EXAMPLES.

1. Reduce $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{7}{8}$, to fractions having the least common denominator possible.

$$\begin{array}{r|rrr} 4 & 3 & 4 & 8 \\ \hline & 3 & 1 & 2 \\ \hline \end{array}$$

$4\times3\times2=24=$least common denominator.

$\overline{24\div3}\times1=8$ the first numerator ; $\overline{24\div4}\times3=18$ the second numerator ; $\overline{24\div8}\times8=21$ the third numerator.

Whence the required fractions are $\frac{8}{24}$, $\frac{18}{24}$, $\frac{21}{24}$.

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$, to fractions having the least common denominator.

Ans. $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$.

CASE VII.

To reduce a fraction of one denomination to an equivalent fraction of a higher denomination.

RULE.—Reduce the given fraction to a compound one by comparing it with all the denominations between it and that denomination you would reduce it to ; lastly, reduce this compound fraction to a single one, by Case V, and you will have a fraction of the required denomination, equal in value to the given fraction.

NOTE.—The reason of the rule may be shown in the following manner :—As there are 12 pence in a shilling, four-fifths of one penny can be only a *twelfth* part as much of 12 pence or a shilling, as it is of one penny. Hence, to reduce four-fifths of a penny to the fraction of a shilling, the given fraction must be diminished 12 times, or one-twelfth of it will be the equivalent fraction of a shilling. And in general, the fraction of one denomination must be as much diminished to be an equivalent fraction of a higher denomination, as is indi-

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160. *How do you reduce a fraction of one denomination to an equivalent fraction of a higher ?*

cated by the *number of parts* of the given denomination which make *one* of the higher denomination.

EXAMPLES.

1. Reduce $\frac{4}{7}$ of a cent to the fraction of a dollar.

By comparing it, it becomes $\frac{4}{7}$ of $\frac{1}{10}$ of $\frac{1}{10}$, which, reduced by Case V., will be

$$\frac{4\times 1\times 1=4}{\text{and } 7\times 10\times 10=700}=\frac{1}{175}\text{ doll. } \textit{Ans.}$$

2. Reduce $\frac{3}{5}$ of a penny to the fraction of a pound.

Ans. £$\frac{1}{400}$.

3. Reduce $\frac{4}{7}$ of an ounce to the fraction of a lb. Avoirdupois.

Ans. $\frac{1}{28}$lb.

4. Reduce $\frac{7}{8}$ of a pennyweight to the fraction of a lb. Troy.

Ans. $\frac{7}{1920}$lb.

CASE VIII.

To reduce a fraction of one denomination to the fraction of another, but less, retaining the same value.

Rule.—Multiply the given numerator by the parts of the denominations between it and that denomination you would reduce it to, for a new numerator, which place over the given denominator: *Or*, only invert the parts contained in the integer, and make of them a compound fraction as before; then reduce it to a simple one.

Note.—This rule is the reverse of the preceding, and the reason of it may be shown in a similar manner. The fraction of a higher denomination is obviously less than the equivalent fraction of a lower denomination; for example, $\frac{3}{5}$ of a pound are $\frac{60}{5}$ of a shilling, or 12 shillings. Whence the value of the fraction must be increased, to render it an equivalent fraction of a lower denomination, so many times as there are parts of the less denomination in a higher.

EXAMPLES.

1. Reduce $\frac{1}{175}$ of a dollar to the fraction of a cent.

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161. *What is the rule for reducing a fraction of one denomination to another less, retaining the same value?*

By comparing the fraction, it will be $\frac{1}{175}$ of $\frac{10}{1}$ of $\frac{10}{1}$; then

$$\frac{1}{175} \times \frac{10}{1} \times \frac{10}{1} = \frac{100}{175} = \frac{4}{7} \text{ c. Ans.}$$

2. Reduce $\frac{1}{400}$ of a pound to the fraction of a penny.
Ans. $\frac{3}{5}$ d.

3. Reduce $\frac{1}{28}$ of a lb. Avoirdupois to the fraction of an ounce.
Ans. $\frac{4}{7}$ oz.

4. Reduce $\frac{7}{1920}$ of a lb. Troy to the fraction of a pwt.
Ans. $\frac{7}{8}$ pwt.

CASE IX.

To find the value of a fraction in the known parts of the integer; as of coin, weight, measure, &c.

Rule.—Multiply the numerator by the parts in the next inferior denomination, and divide the product by the denominator; and if any thing remain, multiply it by the next inferior denomination, and divide by the denominator as before, and so on, as far as necessary; and the quotients placed after one another, in their order, will be the answer required. *Or*, reduce the numerator, as if it were a whole number, to the lowest denomination, and divide the result by the denominator; the quotient will be the number of the lowest denomination, (which must be brought into higher denominations as far as it will go,) and the remainder will be a numerator, to be placed over the given denominator for a fraction of the lowest denomination.

Note.—From this rule, in connexion with what has been said of Reduction of Federal Money, it appears, that, annexing to the given numerator as many ciphers, as will fill all the places to the lowest denomination, and dividing the number so formed by the denominator, the quotient will be the answer in the several denominations, and the remainder a numerator to be placed over the given denominator, forming a fraction of the lowest denomination.

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162. *How do you find the value of a fraction in the known parts of the integer?*

EXAMPLES.

1. What is the value of $\frac{5}{8}$ of a dollar?

By the general rule.

```
  5
  10
  ——
8)50(  2
  ——
$ 6   10
      ——
   8)20(  4
     ——
   c. 2   10
        8)40
          ——
       m. 5
```

By the note.

```
  $  d.  c.  m.
8)5  0   0   0
 ——————————————
     6   2   5
```

Ans. 6d. 2c. 5m.
or 62c. 5m.

Or thus.

$5=5000m. and $\frac{5000}{8}$m.=625m.=62c. 5m. answer, as before.

2. What is the value of $\frac{17}{64}$ of a dollar?

```
    $.  d.  c.  m.
64)17   0   0   0 (2d.  6c.  5 5/8m.
   128
   ———
    420
    384
    ———
     360
     320
     ———
      40
      —— = 5/8
      64
```

or, 26c. $5\frac{5}{8}$m.

Or, $17=17000m.

And $\frac{17000}{64}$m.=$265\frac{5}{8}$m.=26c. $5\frac{5}{8}$m. answer as before.

3. What is the value of $\frac{5}{7}$ of a pound?

Ans. 14s. 3d. $1\frac{5}{7}$qr.

4. What is the value of $\frac{4}{5}$ of a lb. Avoirdupois?

Ans. 12oz. $12\frac{4}{5}$dr.

CASE X.*

To reduce any given quantity to the fraction of any greater denomination of the same kind.

Rule.—Reduce the given quantity to the lowest term mentioned, for a numerator; then reduce the integral part to the same term for a denominator; which will be the fraction required.

Note.—It appears from this rule, and what has been said before, that, in Federal Money, where the given quantity contains no fraction of its lowest denomination, the annexing of as many ciphers to 1 of the required denomination, as will extend to the lowest denomination in the given quantity, will form a denominator, which, placed under the given quantity used as one number for a numerator, will make the answer, which may be reduced to its lowest terms. Or, if there be a fraction of the lowest denomination, multiply the given whole numbers by its denominator, adding its numerator, for a numerator; and let the denominator itself, at the left of as many ciphers as were mentioned above, be a denominator; the fraction so formed will be the answer; which may be reduced to its lowest terms.

EXAMPLES.

1. Reduce 6d. 2c. 5m. to the fraction of a dollar.

By the general rule.

6d.	10d. int. part.
×10+2	10
62	100
×10+5	10
625	1000

And $\frac{625}{1000}=\frac{5}{8}$\$. Ans.

By the note.

\$. d. c. m.

$$\frac{6\ \ 2\ \ 5}{1\ \ 0\ \ 0\ \ 0}=\tfrac{5}{8}\$.$$

Ans., as before

2. Reduce 26c. $5\frac{5}{8}$m. to the fraction of a dollar.

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* This case is the reverse of the former, and therefore proves it.

☞ If there be a fraction given with the said quantity, it must be farther reduced to the denominative parts thereof, adding thereto the numerator.

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163. *What is your rule for reducing any given quantity to the fraction of a greater denomination of the same kind?*

By the general rule.

$$\begin{array}{ll} 26\text{c.} & 100\text{c. int. part.} \\ \times 10+5 & 10 \\ \hline 265 & 1000 \\ \times 8+5 & 8 \\ \hline 2125 & 8000 \end{array}$$

And $\frac{2125}{8000}=\frac{17}{64}$\$. Ans.

By the note.

$$\begin{array}{lcccc} \rule{2em}{0.4pt} & \$. & \text{d.} & \text{c.} & \text{m.} \\ 265\times8+5= & 2 & 1 & 2 & 5 \\ \hline \text{And } \$1.\times8= & 8 & 0 & 0 & 0 \end{array} =\frac{17}{64}\$.$$

Ans. as before.

3. Reduce 14s. 3¼d. $\frac{5}{7}$ to the ſraction of a pound.
Ans. $\frac{4800}{6720}=\frac{5}{7}$£.

4. Reduce 12oz. 12⅘dr. to the ſraction of a lb. Avoirdupois.
Ans. ⅘lb.

* * *

ADDITION OF VULGAR FRACTIONS.

RULE.

Reduce compound fractions to single ones ; mixed numbers to improper fractions ; fractions of different integers to those of the same ; and all of them to a common denominator ; then, the sum of the numerators written over the common denominator will be the sum of the fractions required.

Note.—Fractions, before they are reduced to a common denominator, are entirely dissimilar, and therefore cannot be incorporated with one another ; but when they are reduced to a common denominator, and made parts of the same thing, their sum, or difference, may then be as properly expressed by the sum or difference of the numerators, as the sum or difference of any two quantities whatever, by the sum or difference of their individuals ; whence the reason of the rules, both for Addition and Subtraction, is manifest.

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164. *What is the rule for addition of vulgar fractions ?*

EXAMPLES.

1. Add $\frac{3}{4}$, $\frac{1}{2}$ of $\frac{2}{3}$, 4, and $5\frac{1}{2}$ together.

First, $\frac{1}{2}$ of $\frac{2}{3}=\frac{2}{6}$; $4=\frac{4}{1}$; and $5\frac{1}{2}=\frac{11}{2}$.

Then the fractions are $\frac{3}{4}$, $\frac{2}{6}$, $\frac{4}{1}$, $\frac{11}{2}$.

$$2)\frac{3}{4}\quad\frac{2}{6}\quad\frac{4}{1}\quad\frac{11}{2}.$$

$2\times3\times1\times1\times2=12$ least common denominator.

$$\left.\begin{array}{l} 12\div4=\ 3\times\ 3=\ 9 \\ 12\div6=\ 2\times\ 2=\ 4 \\ 12\div1=12\times\ 4=48 \\ 12\div2=\ 6\times11=66 \end{array}\right\}\text{new numerators.}$$

127 sum of the numerators.

12 least common denominator.

Ans. $\frac{127}{12}=10\frac{7}{12}$.

2. What is the sum of $\frac{7}{16}$ of $4\frac{5}{8}$, $\frac{3}{4}$ of $\frac{1}{3}$, and $9\frac{1}{4}$?

Ans. $12\frac{58}{86}$.

3. Add together $\frac{1}{5}$\$. $\frac{5}{8}$c. $\frac{3}{16}$c. and $\frac{7}{8}$m.

Ans. 20c. 9m.

4. Add £$\frac{1}{9}$. $\frac{3}{7}$s. and $\frac{4}{5}$d. together.

Ans. 2s. $8\frac{64}{105}$d.

5. Add $\frac{1}{4}$ of a week, $\frac{1}{3}$ of a day, $\frac{1}{2}$ of an hour, and $\frac{3}{4}$ of a minute together.

Ans. 2 days, 2 hours, 30 minutes, 45 seconds.

SUBTRACTION OF VULGAR FRACTIONS.

RULE.

PREPARE the fraction as in Addition, and the difference of the numerators, written above the common denominator, will give the difference of the fractions required.*

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* In subtracting mixed numbers, when the fractions have a common denominator, and the numerator in the subtrahend is less than that in the minuend, the difference of the whole numbers will be a whole number, and the difference of the numerators a numerator to be placed over the given denominator; this whole number and the fraction thus formed will be the remainder; but, when the numerator in the subtrahend is greater that that in the minuend subtract the numerator in the subtrahend from the common denominator, adding the numerator in the minuend, and carrying one to the integer of the subtrahend. Hence,

165. *What is your rule for the subtraction of vulgar fractions?*

EXAMPLES.

1. From $\frac{3}{4}$ take $\frac{2}{7}$ of $\frac{5}{8}$.

$\frac{2}{7}$ of $\frac{5}{8}=\frac{10}{56}=\frac{5}{28}$.

Then the fractions are $\frac{3}{4}$ and $\frac{5}{28}$.

3×28= 84
5× 4= 20
4×28=112 com. den.

$\frac{3}{4}=\frac{84}{112}$, and $\frac{5}{28}=\frac{20}{112}$, therefore,
$\frac{84}{112}-\frac{20}{112}=\frac{64}{112}=\frac{4}{7}$ remainder.

2. From $\frac{49}{50}$ take $\frac{5}{9}$. Ans. $\frac{191}{450}$.

3. Take $3\frac{1}{3}$c. from $\frac{1}{5}$ of $2\frac{1}{3}$$. Ans. $43\frac{1}{3}$c.

4. From £$\frac{1}{4}$. take $\frac{9}{10}$s. Ans. 4s. $1\frac{1}{5}$d.

5. From 5 weeks take $19\frac{4}{5}$ days.
Ans. 15da. 4ho. 48min.

MULTIPLICATION OF VULGAR FRACTIONS.

RULE.

Reduce compound fractions to simple ones, and mixed numbers to improper fractions; then the product of the numerators will be the numerator, and the product of the denominators, the denominator of the product required.

Note.—Where several fractions are to be multiplied, if the numerator of one fraction be equal to the denominator of another, their equal numerators and denominators may be omitted.

Hence, a fraction is subtracted from a whole number, by taking the numerator of the fraction from its denominator, and placing the remainder over the denominator, then taking one from the whole number.

EXAMPLES.

From	$12\frac{3}{5}$	$12\frac{2}{5}$	12
Take	$7\frac{2}{5}$	$7\frac{3}{5}$	$\frac{2}{5}$
Rem.	$5\frac{1}{5}$	$4\frac{4}{5}$	$11\frac{3}{5}$

166. *What is the rule for multiplying vulgar fractions?*

EXAMPLES.

1. What is the continued product of $4\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{4}$ of $\frac{7}{8}$, and 6 ?

$$4\tfrac{1}{3}=\tfrac{13}{3},\ \tfrac{1}{4}\text{ of }\tfrac{7}{8}=\frac{1\times7}{4\times8}=\tfrac{7}{32},\text{ and }6=\tfrac{6}{1}.$$

$$\text{Then }\tfrac{13}{3}\times\tfrac{1}{5}\times\tfrac{7}{32}\times\tfrac{6}{1}=\frac{13\times1\times\ 7\times6}{3\times5\times32\times1}=\tfrac{546}{480}=1\tfrac{11}{80}\text{ the answer.}$$

2. Multiply $\frac{4}{17}$ by $\frac{5}{27}$. Ans. $\frac{30}{459}$.

3. Multiply $5\frac{1}{4}$ by $\frac{1}{6}$. Ans. $\frac{7}{8}$.

4. Multiply $\frac{1}{3}$ of 5 by $\frac{3}{4}$ of $\frac{2}{7}$. Ans. $\frac{5}{14}$.

* * *

DIVISION OF VULGAR FRACTIONS.

RULE.

Prepare the fractions as before ; then invert the divisor, and proceed exactly as in Multiplication : The products will be the quotient required.

Note.—To multiply a fraction by an integer, divide the denominator, or multiply the numerator by it; and to divide by an integer, divide the numerator, or multiply the denominator by it.

EXAMPLES.

1. Divide $\frac{1}{3}$ of 17 by $\frac{2}{3}$ of $\frac{6}{8}$.

$$\tfrac{1}{3}\text{ of }17=\tfrac{1}{3}\text{ of }\tfrac{17}{1}=\frac{1\times17}{3\times\ 1}=\tfrac{17}{3},\text{ and }\tfrac{2}{3}\text{ of }\tfrac{6}{8}=\tfrac{12}{24}=\tfrac{1}{2};\ \text{ therefore.}$$

$$\tfrac{17}{3}\div\tfrac{1}{2}=\frac{17\times2}{3\times1}=\tfrac{34}{3}=11\tfrac{1}{3},\text{ the quotient required.}$$

2. Divide $\frac{5}{7}$ by $\frac{3}{5}$. Ans. $1\frac{4}{21}$.

3. Divide $5\frac{1}{8}$ by $7\frac{3}{4}$. Ans. $\frac{41}{62}$.

4. Divide $\frac{3}{7}$ by 9. Ans. $\frac{1}{21}$.

5. Divide $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{2}{3}$ by $\frac{1}{8}$ of $\frac{8}{4}$. Ans. $\frac{8}{9}$.

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167. *What is the rule for the division of vulgar fractions ?*

DECIMAL FRACTIONS.

[Note.—The denominations of Federal Money are purely decimal; dollars being units or whole numbers, dimes tenths of a dollar, cents hundredths of a dollar, and mills thousandths of a dollar; consequently, *Federal Money* and *Decimal Fractions* are subject to the same methods of operation.]

A Decimal Fraction is a fraction whose denominator is a unit with so many ciphers annexed as the numerator has places of figures.

As the denominator of a decimal fraction is always 10, 100, 1000, &c. the denominators need not be expressed: For the numerator only may be made to express the true value. For this purpose it is only required to write the numerator with a point before it at the left hand, to distinguish it from a whole number, when it consists of so many figures, as the denominator hath ciphers annexed to unity, or 1: So $\frac{5}{10}$ is written .5; $\frac{33}{100}$, .33; $\frac{735}{1000}$, .735, &c.

The point prefixed is called a Separatrix.

But if the numerator has not so many places as the denominator has ciphers, put so many ciphers before it, viz. at the left hand, as will make up the defect: So write $\frac{5}{100}$ thus, .05; and $\frac{6}{1000}$, thus, .006, &c. And thus do these fractions receive the form of whole numbers.

We may consider unity as a fixed point, from whence whole numbers proceed infinitely increasing toward the left hand, and decimals infinitely decreasing toward the right hand to 0, as in the following

TABLE.

Tens of Millions.	5	8th place	Whole numbers.
Millions.	5	7th place	
Hundreds of Thousands.	5	6th place	
Tens of Thousands.	5	5th place	
Thousands.	5	4th place	
Hundreds.	5	3rd place	
Tens.	5	2nd place	
Units.	5	1st place	
	·	.	
Tenths.	5	1st place	Decimal parts.
Hundredths.	5	2nd place	
Thousandths.	5	3rd place	
Ten-Thousandths.	5	4th place	
Hundred-Thousandths.	5	5th place	
Millionths.	5	6th place	
Ten-Millionths.	5	7th place	
Hundred-Millionths.	5	8th place	

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168. *What are Decimal Fractions?*——169. *Explain the principles on which decimals are founded?*——170. *What is said of unity?*——171. *Explain* the Table.

All the figures at the left hand of the decimal point are whole numbers. The 5 in the 1st place at the right hand of the point, represents 5 tenths ; 5 in the 2nd place, 5 hundredths ; 5 in the 3rd place, 5 thousandths ; 5 in the 4th place, 5 ten-thousandths ; 5 in the 5th place, 5 hundred thousandths ; 5 in the 6th place, 5 millionths ; 5 in the 7th place, 5 ten-millionths ; 5 in the 8th place, 5 hundred-millionths ; and all of them taken together, are read thus :—Fifty-five million, five hundred fifty-five thousand, five hundred and fifty-five hundred millionths.*

Ciphers, placed at the right hand of a decimal fraction, do not alter its value, since every significant figure continues to possess the same place : So .5, .50, and .500, are all of the same value, and each equal to $\frac{1}{2}$.

But ciphers placed at the left hand of a decimal, do alter its value, every cipher depressing it to $\frac{1}{10}$ of the value it had before, by removing every significant figure one place further from the place of units. So .5, .05, .005, all express different decimals, viz. .5, $\frac{5}{10}$; .05, $\frac{5}{100}$; .005, $\frac{5}{1000}$.

Decimal fractions of unequal denominators are reduced to one common denominator, when there are annexed to the right hand of those, which have fewer places, so many ciphers as make them equal in places with that which has the most. So these decimals, .5, 06, .455, may be reduced to the decimals .500, .060, .455, which have all 1000 for their denominator.

Of decimals, that is the greatest, whose highest figure is greatest, whether they consist of an equal or unequal number of places. Thus, .5 is greater than .459, for if it be reduced to the same denominator with .459, it will be .500.

---

*It is evident from the Table, that since the decimal parts *decrease* in a tenfold proportion from the place of units towards the right hand, they must *increase* in a tenfold proportion from the right hand towards the left, which proves that decimal fractions are subject to the same law of notation, and consequently of operation, as whole numbers.

To read decimal figures.—Begin at the left hand, and read them in the same manner as we read whole numbers, and add to the whole the name of the place of the right hand figure.

EXAMPLES.

.24 is read 24 hundredths ;
.035 is read 35 thousandths ;
.0050 is read 50 ten thousandths ;
.000750 is read 750 millionths ;
.00000099 is read 99 hundred millionths.

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172. *What are all figures at the left hand of the point, or separatrix ?*——173. *What does the first figure at the right of the decimal point represent ?*——174. *What the second, third. fourth, &c. ?*——175. *What effect have ciphers placed on either hand of a decimal ?*——176. *How are decimal fractions of unequal denominators reduced to a common denominator ?*

A mixed number, viz. a whole number with a decimal annexed, is equal to an improper fraction, whose numerator is all the figures of the mixed number, taken as one whole number, and the denominator that of the decimal part. So 45·309 is equal to $\frac{45309}{1000}$, as is evident from the method given to reduce a mixed number to an improper fraction : Thus, 45×1000+.309=$\frac{45309}{1000}$ as above.

◆

ADDITION OF DECIMALS.

RULE.

1. Place the numbers, whether mixed, or pure decimals, under each other, according to the value of their places.

2. Find their sum, as in whole numbers, and point off so many places for decimals, as are equal to the greatest number of decimal places in any of the given numbers.

EXAMPLES.

1. Find the sum of 19·073+2·3597+223+·0197581+3478·1+12·358.

```
  19·073
   2·3597
 223.
    ·0197581
3478·1
  12.358
―――――――――――――
3734·9104581 the sum.
```

2. Required the sum of 429+21·37+355·003+1·07+1·7 ?
Ans. 808·143

3. Required the sum of 973+19+1·75+93·7164+·9501 ?
Ans. 1088·4165.

4. Required the sum of 5·3+11·973+49+·9+1·7314+34·3 ?
Ans. 103·2044.

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177. *What is the rule for addition of decimals ?*

SUBTRACTION OF DECIMALS.

RULE.

Place the numbers according to their value: Then subtract as in whole numbers, and point off the decimals as in Addition.

EXAMPLES.

1. Find the difference of 1793·13 and 817·05693.

From 1793·13
Take 817·05693

Remainder 976·07307

2. From 171·195 take 125·9176. Ans. 45.2774.

3. From 219·1384 take 195·91. Ans. 23·2284.

4. From 480 take 245·0075. Ans. 234·9925.

* * *

MULTIPLICATION OF DECIMALS.

RULE.

1. Whether they be mixed numbers, or pure decimals, place the factors and multiply them as in whole numbers.

2. Point off so many figures from the product as there are decimal places in both the factors; and if there be not so many places in the product, supply the defect by prefixing ciphers.

NOTE.—The reason of the rule for pointing off the figures for decimals, is evident from the notation of decimals. Thus, ·5×·5=·25; for ·5×1=·5 or once-five tenths. But as the multiplier is less than unity, or tenths, multiplying is only taking tenths of tenths, and so many tenths of tenths, are evidently so many hundredths. So also, tenths of hundredths would be thousandths; hundredths of hundredths would be ten thousandths; and so on.

---

178. *What is the rule for subtracting decimal fractions?*——179. *What is the rule for multiplication of decimals?*

EXAMPLES.

$$\begin{array}{lr} \text{1. Multiply} & .02345 \\ \text{by} & .00163 \\ \hline & 7035 \\ & 14070 \\ & 2345 \\ \hline & .0000382235 \text{ the product.} \\ \hline \end{array}$$

2. Multiply 25.238 by 12.17. Ans. 307.14646.

3. Multiply .3759 by .945. Ans. .3552255.

4. Multiply .84179 by .0385. Ans. .032408915.

To multiply by 10, 100, 1000, &c. remove the separating point so many places to the right hand, as the multiplier has ciphers.

So .345 multiplied by $\left\{\begin{array}{l}10\\100\\1000\end{array}\right\}$ makes $\left\{\begin{array}{r}3.45\\34.5\\345.\end{array}\right.$
For .345×10 is 3.450, &c.

DIVISION OF DECIMALS.

RULE.

1. The places of decimal parts in the divisor and quotient, counted together, must always be equal to those in the dividend; therefore, divide as in whole numbers, and, from the right hand of the quotient, point off so many places for decimals, as the decimal places in the dividend exceed those in the divisor.

2. If the places of the quotient be not so many as the rule requires, supply the defect by prefixing ciphers to the left hand.

3. If at any time there be a remainder, or the decimal places in the divisor be more than those in the dividend, ciphers may be annexed to the dividend or to the remainder, and the quotient carried on to any degree of exactness.

---

180. *What is the rule for division of decimals?*

EXAMPLES.

1.

219).117841075(.000538087, &c.
1095
834
657
1771
1752
1907
1752
1555
1533
22

2.

.3719)38.0000(102.178, &c.
3719
8100
7438
6620
3719
29010
26033
29770
29752
18

☞ In Example 1st, the divisor having no decimals, the quotient must have so many as there are in the dividend. In Example 2d, the dividend being an integer, must have at least so many ciphers annexed, as there are decimals in the divisor, and so far the quotient will be whole numbers; then annexing more ciphers, the remaining figures in the quotient will be decimals, according to the rule.

3. 133)5737(43.1353+*

4. 23.7)65321(2756.16+

5. †72)918.217(12753+

6. †25.17)315.6293(1253+

7. †.317)29.417(92+

When decimals or whole numbers are to be divided by 10, 100, 1000, &c. [viz. unity with ciphers] it is performed by removing the separatrix, in the dividend, so many places toward the left hand, as there are ciphers in the divisor.

$$\text{So}\left\{\begin{matrix}10\\100\\1000\\10000\end{matrix}\right\}\text{dividing 7654, the quotient is}\left\{\begin{matrix}765.4\\76.54\\7.654\\.7654\end{matrix}\right.$$

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* This character, placed here, shews that the quotient is not complete, and that it may be further continued by annexing ciphers to the remainder.

† These questions are left unpointed in the quotient, to exercise the learner.

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181. *When decimals or whole numbers are to be divided by* 100, 1000, *&c. how do you proceed?*

REDUCTION OF DECIMALS.

CASE I.

To reduce a vulgar fraction to its equivalent decimal.

RULE.

Divide the numerator by the denominator, as in Division of Decimals, and the quotient will be the decimal required. Or, so many ciphers as you annex to the given numerator, so many places must be pointed off in the quotient, and if there be not so many places of figures in the quotient, the deficiency must be supplied by prefixing so many ciphers before the quotient figures.

Note.—By annexing one, two, three, &c. ciphers to the numerator, the value of the fraction is increased ten, a hundred, &c. times. After dividing, the quotient will of course be ten, a hundred, &c. times too much; the quotient must therefore be divided by ten, a hundred, &c. to give the true quotient or fraction. In the first example, $\frac{1}{8}$ is made $\frac{1000}{8}=\frac{125}{1}$, which, divided by 1000, is $\frac{125}{1000}=.125$, and is the rule.

EXAMPLES.

1. Reduce $\frac{1}{8}$ to a decimal.

$$\begin{array}{r} 8)\underline{1{\cdot}000} \\ {\cdot}125 \text{ Ans.} \end{array}$$

2. Reduce $\frac{3}{8}$, $\frac{5}{8}$, $\frac{2}{3}$ to decimals.
Ans. ·375, ·625, ·666+.

3. Reduce $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{7}{8}$ to decimals.
Ans. ·25, ·5, ·75, ·333+, ·8, ·833+, ·875.

4. Reduce $\frac{5}{19}$, $\frac{27}{39}$, $\frac{12}{480}$, and $\frac{9}{36}$ to decimals.
Ans. ·263+, ·692+, ·025, ·25

CASE II.

To reduce numbers of different denominations, as of Money, Weight, and Measure, to their equivalent decimal fractions.

RULE.

1. Write the given numbers perpendicularly under each other, for dividends, proceeding orderly from the least to the greatest.

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182. *How do you proceed to reduce a vulgar fraction to its equivalent decimal?*——183. *What is your rule for reducing numbers of different denominations to their equivalent decimal value?*

2. Opposite to each dividend, on the left hand, place such a number, for a divisor, as will bring it to the next superior denomination, and draw a line perpendicularly between them.

3. Begin with the highest, and write the quotient of each division as decimal parts on the right hand of the dividend next below it, and so on, until they are all used, and the last quotient will be the decimal sought.

NOTE.—The reason of the rule may be explained from the first example: thus, three farthings are $\frac{3}{4}$ of a penny, which, brought to a decimal, is ·75; consequently, $8\frac{3}{4}$d. may be expressed, 8·75d. but 8·75 is $\frac{875}{100}$ of a penny $=\frac{875}{1200}$ of a shilling, which, brought to a decimal, is ·729166s.+ In like manner, 17·729166s.+ are $\frac{17729166}{1000000}$s. $=\frac{17729166}{20000000}$£ $=$ ·886458+ as by the rule.

EXAMPLES.

1. Reduce 17s. $8\frac{3}{4}$d. to the decimal of a pound.

4	3.
12	8.75
20	17.729166, &c.
	·886458, &c. the decimal required.

☞ Here, in dividing 3 by 4, we suppose two ciphers to be annexed to the 3, which make it 3·00, and ·75 is the quotient, which we write against 8 in the next line; this quotient, viz. 8·75, being pence and decimal parts of a penny, we divide them by 12, which brings them to shillings and decimal parts; we therefore divide by 20, and (there being no whole number) the quotient is decimal parts of a pound.

2. Reduce 1, 2, 3, 4, and so on to 19 shillings, to decimals.

Shillings.	1	2	3	4	5	6
Answers.	·05,	·1,	·15,	·2,	·25,	·3,

Shillings.	7	8	9	10	11	12
Answers.	·35,	·4,	·45,	·5,	·55,	·6,

Shillings.	13	14	15	16	17	18	19
Answers.	·65,	·7,	·75,	·8,	·85,	·9,	·95.

☞ Here, when the shillings are even, half the number, with a point prefixed, is their decimal expression; but if the number be odd, annex a cipher to the shillings, and then halving them, you will have their decimal expression.

3. Reduce 1, 2, 3, and so on to 11 pence, to the decimals of a shilling.*

Pence.	1	2	3	4	5	6
Answers.	.083+,	.166,	.25,	.333+,	.416+,	.5,

Pence.	7	8	9	10	11
Answers.	.533+,	.666+,	.75,	.833+,	.916+.

4. Reduce 1, 2, 3, &c. to 11 pence, to the decimals of a pound.

Pence.	1	2	3	4
Answers.	.00416+,	.0083+,	.0125,	.01666+,

Pence.	5	6	7	8
Answers.	.0208+,	.025,	.02916+,	.0333+,

Pence.	9	10	11
Answers.	.0375,	.0416+,	.04583+,

CASE III.

To find the decimal of any number of shillings, pence, and farthings, by inspection.

RULE.

1. Write half the greatest even number of shillings for the first decimal figure.

2. Let the farthings in the given pence and farthings, possess the second and third places; observing to increase the second place, or place of hundredths, by 5, if the shillings be odd, and the third place by 1, when the farthings exceed 12, and by 2 when they exceed 36.

NOTE.—If there be no shillings, or only one shilling in the given sum, a cipher must be written in the *first* place, or place of tenths.

If the farthings in the given pence and farthings, do not exceed 9, a cipher must be written in the *second* place, or place of hundredths.

The invention of the rule is as follows:—As shillings are so many 20ths of a pound, half of them must be so many tenths, and consequently take the place of tenths in the decimals; but when they are odd, their half will always consist of two figures, the first of which will be half of the even number, next less, and the second a 5: Again, farthings are 960ths of a pound, and had it happened that 1000, instead of 960, had made a pound, it is plain any number of farthings would have made so many thousandths, and might have taken their place in the decimal accordingly. But 960 increased by $\frac{1}{24}$ part of

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* The answers to this question are the same as the decimal parts of a foot.

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184. *What is the rule for finding the decimal of any number of shillings, pence, &c., by inspection?*

itself, is =1000, consequently any number of farthings, increased by their $\frac{1}{24}$ part, will be an exact decimal expression for them: Whence, if the number of farthings be more than 12, $\frac{1}{24}$ part is greater than ½qr. and, therefore, 1 must be added; and when the number of farthings is more than 36, $\frac{1}{24}$ part is greater than 1½qr. for which 2 must be added.

EXAMPLES.

1. Find the decimal of 13s. 9¾d. by inspection.

.6 . . =½ of 12s.
5 for the odd shilling.
39 = farthings in 9¾d.
Add 2 for the excess of 36.

.691 =decimal required.

2. Find by inspection the decimal expressions of 18s. 3¼d. and 17s. 8½d. Ans. £.914 and £.885.

CASE IV.

To find the value of any given decimal in the terms of the integer.

RULE.—1. Multiply the decimal by the number of parts in the next less denomination, and cut off so many places for a remainder, to the right hand, as there are places in the given decimal.

2. Multiply the remainder by the next inferior denomination, and cut off a remainder as before.

3. Proceed in this manner through all the parts of the integer—and the several denominations, standing on the left hand, make the answer.

EXAMPLES.

1. Find the value of .73968 of a pound.

```
     20
 --------
 14.79360
       12
 --------
  9.52320
        4
 --------
  2.09280     Ans. 14s. 9½d.
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185. *What is the rule for finding the value of any given decimal in the terms of the integer?*

2. What is the value of .679 of a shilling? Ans. 8.148d.

3. What is the value of .617 of a cwt.? Ans. 2qrs. 13lb. 1oz. $10\frac{6}{10}$dr.

4. What is the value of .397 of a yard? Ans. 1qr. 2.352n.

5. What is the value of .8469 of a degree? Ans. 58m. 6fur. 35po. 0ft. 11in.

6. What is the value of .569 of a year? Ans. 207d. 16h. 26m. 24sec.

COMPOUND MULTIPLICATION.

COMPOUND MULTIPLICATION is the multiplying of sums of different denominations; it is useful in finding the value of Goods, &c. And, as in compound Addition, we carry from the lowest denomination to the next higher, so we begin and carry in Compound Multiplication; one general rule being to multiply the price by the quantity.

NOTE.—The product of a number, consisting of several parts or denominations, by any simple number whatever, will be expressed by taking the product of that simple number, and each part by itself, as so many distinct questions: Thus, 33l. 15s. 9d. multiplied by 5, will be 165l. 75s. 45d.=(by taking the shillings from the pence, and the pounds from the shillings, and placing them in the shillings and pounds respectively,) 168l. 18s. 9d. and this will be true when the multiplicand is any compound number whatever.

CASE I.

When the multiplier or quantity does not exceed 12.

Multiply the price of one yard, pound, &c. by the whole quantity or number of yards, pounds, &c.—the product will be the answer.

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166. What is Compound Multiplication?——167. What is the rule when the multiplier does not exceed 12?

EXAMPLES.

1. What will 5 yards of broadcloth amount to, at £1 12s. 4½d. a yard?

£	s.	d.
1	12	4½
		5
£8	1	10½

☞ In this example, we write down £1. 12s. 4½d. the price of one yard, and then write 5, the number of yards, under the least denomination. We multiply 2 farthings by 5, and the product is 10 farthings, which we bring into pence by dividing them by 4; we write down the remaining 2 farthings, and reserve the quotient, 2 pence, to be added to the product of the pence. We then multiply 4 pence by 5, and the product is 20 pence, and 2 pence which we reserved are 22 pence, which we bring into shillings by dividing them by 12; we write down the remainder 10 pence, and reserve the quotient, 1 shilling, to be added to the product of the shillings. We then multiply 12 shillings by 5, and the product is 60 shillings, and 1 shilling that we reserved are 61 shillings, which we bring into pounds by dividing them by 20; we write down the remainder 1 shilling, and reserve the quotient, 3 pounds, to be added to the product of the pounds. We then multiply 1 pound by 5, and the product is 5 pounds, and 3 pounds which we reserved are 8 pounds; this being the highest denomination, we write down the whole amount 8 pounds, and find the product or answer to be £8 1s. 10½d.

2. Multiply £4 13s. 4¾d. by 10.

3. Multiply £8 15s. 11¾d. by 11.

4. Multiply £13 12s. 11d. by 7.

5. Multiply £14 17s. 8¼d. by 9.

Ans. £133. 19s. 2¼d.

Remark.—The facility of reckoning in *Federal Money*, compared with *pounds*, *shillings*, &c. may be seen from the examples given in this and the following cases.

The general rule is—

Multiply as in simple mutiplication, and from the product point off so many places for cents and mills, as there are places of cents and mills in the price.

In Pounds, Shillings, &c.

1. What will 9 yards of cloth amount to, at 5s. 4d. per yard ?

	£	s.	d.
Price yer yard,	£0	5	4
Number yards,			9
	£2	8	0

2. Six yards at 9s. 10d. per yard ?
Ans. £2 19s.

In Dollars, Cents and Mills.

1. What will 9 yards of cloth amount to, at 88c. 9m. per yard ?

Price,	88c.	9m.
		9
$8 ·00		1

2. Six yards at $1·22c. 9m. per yard ?
Ans. $7·37c. 4m.

CASE II.

Where the multiplier, that is, the quantity, is above 12.

Multiply by two such numbers, as, when multiplied together, will produce the given quantity, or multiplier.

EXAMPLES.

In Pounds, Shillings, &c.

1. What will 144 yards of cloth cost, at 3s. 5½d. per yard ?

	£	*s.*	*d.*	
	0	3	5½	price 1 yd.
Mult. by			12	
Produces	2	1	6	price 12 yds.
Mult. by			12	
	£24	18	0	price 144 yds.

2. 24 yds. at 6s. 3¾d. ?
Ans. £7 11s. 6d.

3. 27 yds. at 9s. 10d.* ?

In Dollars, Cents and Mills.

1. What will 144 yards of cloth cost, at 57c. 6⅖m. per yard ?

$0·5764	Or,	·5764
144		12
23056		69168
23056		12
5764		$83·0016
$83·0016		

2. 24 yards at $1, 5c. 2m. ?
Ans. $25, 24c. 8m.

3. 27 yds. at $1, 22c. 9m.

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* New-York currency, $1=8s.

188. *What is the rule when the multiplier exceeds* 12 ?

CASE III.

When the quantity is such a number, that no two numbers in the Table will produce it exactly.

Multiply by two such numbers as come the nearest to it; and for the number wanting, multiply the given price of 1 yard by the said number of yards wanting, and add the products together for the answer; but if the product of the two numbers exceeds the given quantity, then find the value of the overplus, which subtract from the last product, and the remainder will be the answer.

EXAMPLES.

In Pounds, Shillings, &c.

1. What will 47 yards of cloth, at 17s. 9d. per yard, amount to?

	£.	s.	d.	
	0	17	9	price of 1 yd.
Mult. by			5	
Produces	4	8	9	price of 5 yds.
Mult. by			9	
Prod.	39	18	9	price of 45 yds.
Add	1	15	6	price of 2 yds.
Ans.	£41	14	3	price 47 yds.

In Dollars, Cents and Mills.

1. What will 47 yards of cloth, at $2 95c. 8m. per yard, amount to?

	$2·958
	47
	20706
	11832
Ans.	$139·026

Note.—This may be performed by first finding the value of 48 yards, from which if you subtract the price of 1, the remainder will be the answer as above.

In Pounds, Shillings, &c.	In Dollars, Cents and Mills.
2. 67½ yards at 16s. 4d.* ? Ans. £55 2s. 6d.	2. 67½ yards at $2 4c. ? Ans. $137 81c.
3. 59 yards at 10s.† ? Ans. £29 10s.	3. 59 yards at $1 33⅓c. ? Ans. $78 66c. 6m.

* New-York currency.

† Pennsylvania currency; $1=7s. 6d.

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189. *How do you proceed when the multiplier or quantity is such a number as no two numbers in the Multiplication Table will produce exactly?*

CASE IV.

When the quantity is any number above the Multiplication Table.

Multiply the price of 1 yard by 10, which will produce the price of 10 yards: *This* product, multiplied by 10, will give the price of 100 yards; then you must multiply the price of 100 by the number of hundreds in your question; the price of ten by the number of tens; and the price of unity, or 1, by the number of units: lastly, add these several products together, and the sum will be the answer.

EXAMPLES.

1. What will 359 yards of cloth, at 4s. 7½d. per yard, amount to?

		s.	*d.*	
		4	7½	price of 1 yard.
			10	
	£2	6	3	price of 10 yards.
			10	
	23	2	6	price of 100 yards.
			3	
	69	7	6	price of 300 yards.
5 times the price of 10 yds.=	11	11	3	price of 50 yards.
9 times the price of 1 yd.=	2	1	7½	price of 9 yards.
Ans.	£83	0	4½	price of 359 yards.

2. 297 yards at 17s. 8½d. per yard?

Ans. £256 15s. 7½d.

3. 297 yards at $2 88c. 2m. per yard?

Ans. $855 95c. 4m.

CASE V.

To find the value of a hundred weight or 112 pounds, having the price of one pound given.

Multiply the price of 1 pound by 7, the product will be the price of 7 pounds; multiply the price of 7 pounds by 4, the pro-

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190. *When the quantity is a number above the Multiplication Table, how do you proceed?*——191. *What is the method of finding the value of a cwt. or 112 lbs. the price of 1 pound being given?*

duct will be the price of 28 pounds, or 1 qr. of a cwt. ; then multiply the price of 28 pounds by 4, and the product will be the price of 112 pounds, or a cwt.

EXAMPLES.

1. What will 1 cwt. of lead come to, at 6½d. per pound?

6½d. price of 1 lb.
7

3 9½ price of 7 lbs.
4

15 2 price of 28 lbs. or ¼ cwt.
4

£3 0 8 price of 1 cwt.

2. 1 cwt. of tin at 2¼d. per lb.
Ans. £1 1s. 0d.

3. 1 cwt. of chalk at 1½d. per lb.?
Ans. 14s.

4. 1 cwt. of chalk at 2c. 1m. per lb.?
Ans. $2 35c. 2m.

PRACTICAL QUESTIONS IN WEIGHTS AND MEASURES.

1. What is the weight of 4 hogsheads of sugar, each weighing 7 cwt. 3qrs. 19lb.? Ans. 31cwt. 2qrs. 20lb.

2. What is the weight of 6 chests of tea, each weighing 3cwt. 2 qrs. 9lb.? Ans. 21cwt. 1qr. 26lb.

3. If I am possessed of 1½ dozen of silver spoons, each weighing 3oz. 5pwt.—2 dozen of tea spoons, each weighing 15pwt. 14gr.—3 silver cans, each 9oz. 7pwt.—2 silver tankards, each 21oz. 15pwt.—and 6 silver porringers, each 11oz. 18pwt.; what is the weight of the whole? Ans. 18lb. 4oz. 3pwt.

4. In 35 pieces of cloth, each measuring 27¾ yards, how many yards? Ans. 971¼ yards.

5. How much brandy in 9 casks, each containing 45 gallons, 3qt. 1pt.? Ans. 412gal. 3qt. 1pt.

6. If I have 9 fields, each of which contains 12 acres, 2 roods, and 25 poles; how many acres are there in the whole?
Ans. 113ac. 3r. 25p.

COMPOUND DIVISION.

Compound Division is the dividing of numbers of different denominations: in doing which, always begin at the highest, and when you have divided that, if any thing remain, reduce it to the next lower denomination, and so on, till you have divided the whole, taking care to set down your quotient figures under their respective denominations.*

INTRODUCTORY EXAMPLES.

	£	s.	d.	
1. Divide	549	17	9	by 5
	£109	19	6¼	

☞ In this example, having divided the pounds, the 4 which remains is £4, which are equal to 80s. and 17 in the shillings, make 97s.; we then find that 5 is contained 19 times in 97, and 2 over: we set down the 19 under the shillings, and reduce the 2s. that remain into pence, and they make 24, which, added to the 9d. in the question, make 33d.;—then how often 5 in 33? 6 times, and 3 over.—we set down 6 under the pence, and reduce the 3d. which remain to farthings—they make 12: 5 is contained in 12 twice; we therefore set down ½d. and the 2 which remains, being ⅖ of a farthing, is disregarded.

2

	T.	*cwt.*	*qrs.*	*lb.*	*oz.*	*dr.*
3)	29	13	2	25	1	13

3

	lb.	*oz.*	*pwt.*	*gr.*
10)	849	11	12	14

4. Divide 58l. 19s. 11½d. by 11.

5. Divide 37l. 17s. 4¾d. by 7.

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* To divide a number, consisting of several denominations by a simple number, is, evidently, the same as dividing all the parts of which it is composed by the same simple number. And this will be true, when any of the parts are not an exact multiple of the divisor; for if the excess of that multiple has its proper value in the next less denomination, the dividend will still be divided into parts, and the true quotient will be found as before. Thus, £8 1s. 10½d. divided by 5, will be the same as £5 60s 20d. 10qr. divided by 5, which is equal to £1 12s. 4½d as by the rule.

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192. *What is Compound Division?*

CASE I.*

Having the price of any number of yards, &c. within the pence table, to find the price of 1 yard, &c.

If the quantity does not exceed 12, proceed by setting down the price, and dividing it by the quantity; which quotient will be the price of one yard, required; but if the quantity exceed 12, then divide by two such numbers as, when multiplied together, will produce the quantity, and the last quotient will be the value of 1 yard, required.

EXAMPLES.

In pounds, shillings, &c.

1. If 9 yards of cloth cost £4 3s. 7½d. what is it per yard?

	£	s.	d.	
9)	4	3	7½	
	0	9	3½	Ans.

2. If 84 cows cost £253 13s. what is the price of each?

Ans. £3 0s. 4⅔d.

In dollars, cents, and mills.

1. If 9 yards of cloth cost $13 93¾c. what is it per yard?

$
9)13.9375

1.5486+Ans.

2. If 84 cows cost $845 50c. what is the price of each?

Ans. $10 6½c.

Note.—If there be a remainder after the division by one of the parts of a composite number before the last, that remainder must be divided according to the rule for division of fractions. Thus,

If 35 yards cost £37 11s. what is the price of 1 yard?

		£	s.		
35=5×7	5)	37	11		
	7)	7	10	2	1qr$\frac{3}{5}$.
		1	1	5	1$\frac{33}{35}$qr. Ans.

CASE II.

Having the price of 1cwt. or 112lbs. to find the price of one pound.

Divide the price of 1 cwt. by 8, that quotient by 7, and this quotient by 2, and the last quotient will be the price of *one pound*, required. The *reason* is, $8 \times 7 \times 2 = 112$ lbs. or 1 cwt.

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* This case proves the 1st and 2d cases in Compound Multiplication.

193. *Having the price of several yards, how do you find the price of one?*——194. *In case of a remainder, how do you proceed?*——195. *Having the price of 1cwt. or more, how do you find that of a pound?*

EXAMPLES.

1. If 1 cwt. of flax cost £2 7s. 8d. what is that per lb.?

	£	s.	d.	
8)	2	7	8	
7)	0	5	11½	
2)	0	0	10	0$\frac{6}{7}$qr.
	0	0	5$\frac{3}{28}$d. price of 1 pound.	

2. At $156 per cwt. what is the price per lb.?
Ans. $1 39$\frac{2}{7}$c.

3. At $3 33c. 3m. per cwt. what is the cost of 1 lb.?
Ans. 2c. 9$\frac{85}{112}$m.

4. If 1 cwt. of sugar cost £3 7s. 6d. what is that per lb.?
Ans. 7¼d.

CASE III.*

Having the price of several hundred weight, to find the price per pound.

Divide the whole price by the number of hundreds, which will give the price per cwt. and then proceed as in the last case.

EXAMPLES.

1. If 5 cwt. sugar cost £13 8s. 4d. what is that per lb.?

	£	s.	d.	
5)	13	8	4	
8)	2	13	8	price of 1 cwt.
7)	0	6	8½	price of 14lb. or $\frac{1}{8}$ cwt.
2)	0	0	11½	price of 2lb. or $\frac{1}{56}$ cwt.
	0	0	5¾	price of 1lb.

---

* This case proves the 6th in Compound Multiplication.

2. If 3cwt. of raisins cost $50 52c. what cost 1lb. ?

$
3)50·52

8)16·84

7)2·05

2)·300$\frac{5}{7}$

·15c.$\frac{5}{14}$m. Ans.

3. If 3$\frac{1}{4}$cwt. cost $3.64, what cost 1lb. ?

Ans. 1c.

4. If 1$\frac{3}{4}$ cwt. cotton wool cost £6 10s. 8d. what is that per lb. ?

5. If 3$\frac{3}{4}$ cwt. cost $17 56c. what is the cost of 1 lb. ?

6. If 11$\frac{1}{2}$ cwt. cost $87 33c. what cost 1 lb. ?

CASE IV.*

Having the price of any number of yards, &c. to find the price of 1 yard.

Divide the price by the quantity, beginning at the highest denomination, and if any thing remain, reduce it into the next, and every inferior denomination, and at each reduction divide as before, remembering each time to add the odd shillings, pence, &c. if there be any, and you will have the value of unity required.

NOTE.—If there be $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$ of a yard, pound, &c. multiply both the price and quantity by 4, and then proceed as above directed; or, in *Federal Money*, work by decimals.

EXAMPLES.

1. If 95$\frac{1}{2}$lb. of figs cost $\left\{\begin{matrix}\text{16l. 13s. 6}\frac{3}{4}\text{d.}\\ \text{\$55 59c. 3}\frac{3}{4}\text{m.}\end{matrix}\right\}$ what are they per lb.?

* This case proves the 3d and 4th cases in Reduction.

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* 196. *Having the price of any number of yards, how do you find the price of one ?*

Quantity = 95½lb.	Price = £16 13 6¾
Mult. by 4	4
Produces 382 for a divisor.	Product £66 14 3 for a dividend.

382)66 14 3(0 3 5¾ $\frac{250}{382}$ per lb.
20

382)1334(3
1146

188
12

382)2259(5
1910

349
4

382)1396(3
1146

250

D.c.m.dec.c. m.
95½=95.5)55.59375(.58 2+Ans.
4775

7843
7640

2037
1910

1275

2. Bought 33¼ yds. cloth for $85 63c. 2m.; what did I pay a yd.?
Ans. $2 57c. 5m.

3. If 33¼ yds. baize cost £25 13s. 9½d. what is it per yard?
Ans. 15s. 5¼d. $\frac{95}{133}$.

PRACTICAL QUESTIONS.

1. Divide $\left\{ \begin{array}{l} £273\ \ 9s.\ \ 4d. \\ \$911\ \ 55c.\ \ 5m. \end{array} \right\}$ among 5 men and 4 women, and give the men twice as much as the women.

Men. Wom.
5 and 4
Mult. by 2

10 shares
Add 4 women's shares.

14 number of equal shares in the whole = Divisor.

```
                 £  s. d.   £   d.  s
Divide by 14)273  9  4(  19  10  8=1 woman's share.
             14                  4
             ——          ————————————
             133          78   2  8= women
             126         ————————————
             ——           19  10  8
               7                  2
              20         ————————————
             ——           39   1  4=1 man's share.
          14)149(10               5 men.
             14          ————————————
             ——          195   6  8=men's share.
               9          78   2  8=women's share.
              12         ————————————
             ——         £273   9  4 Proof.
          14)112(8       ————————————
             112
```

```
   $
14)911.555(65.111+ =1 woman's share.
   84          4 women.
   ——       ——————
   71       260.444=women's share.
   70       ——————
   ——
    15            65.111+
    14                 2
    ——            ———————
     15           130.222+ =1 man's share
     14                 5
     ——           ———————
      15          651.111+ = men's share.
      14          260.444+ = women's share.
      —           911.555+ Proof.
       1
```

2. A man purchased a farm containing 113A. 3r. 25p.—and he sold one ninth part of it; how many acres did he sell?

Ans. 12A. 2r. 25p.

3. Divide 21 bar. 30 gals. 2 qts. by 4.

Ans. 5 bar. 15 gals. 2 qts.

4. A merchant purchased 9 hogsheads of rum, containing 1.039 gallons 2 qts.; what number of gallons did each hogshead contain?

Ans. 115 gals. 2 qts.

5. Divide 45 tons of round timber by 8.

Ans. 5T. 25ft.

6. A man purchased a freight of wood containing 93 cords, and piled it into six ranges of equal dimensions; what was the number of cords in each range?

Ans. 15C. 64ft.

7. If the earth perform 5 revolutions round the sun in 1·826d. 5h. 4m. 45s., in what time will it perform one revolution?

Ans. 365d. 5h. 48m. 57sec.

8. Divide 60° 2′ by 6.

Ans. 10° 0′ 20″.

* * *

INTEREST.

Interest is a premium allowed for the loan of money.

It is estimated at a certain number of dollars, pounds, &c. for each hundred dollars, pounds, &c. for one year; and in the same proportion for a greater or less sum, or for a longer or shorter time. Hence, interest is said to be so much per cent. or per centum, per annum.

The terms used in interest are *principal*, *rate*, and *amount*.

The *principal* is the sum of money lent.

The *rate* is the sum allowed for the use of one hundred dollars or pounds for one year.*

The *amount* is the sum of the principal and interest.

Interest is of two kinds, *Simple* and *Compound*.

SIMPLE INTEREST.

Simple Interest is when a premium is allowed for the principal only.

Note.—Commission, Insurance, Purchasing Stocks, or any thing else, rated at so much per cent. are calculated by the Rules for Simple Interest.

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* Lawful or legal interest is that which is allowed by law. In England, the rate is *five* per cent. In the New-England States, it is *six*; and in the state of New-York, it is *seven* per cent.

197. *What is interest?*——198. *How is it estimated?*——199. *What terms are used in interest?*——200. *What is the principal?*——201. *What the rate; and amount?*——202. *What is Simple Interest?*

GENERAL RULE.

1. Multiply the principal by the rate, and divide the product by 100, the quotient will be the interest for one year.

2. To find the interest for more years than one; multiply the interest of one year by the given number of years, and the product will be the interest for that number of years.

3. To find the interest for parts of a year, as months and days; for the months, take aliquot parts of the interest for one year; and for the days, take aliquot parts of the interest for one month, allowing 30 days to the month.

TABLE OF ALIQUOT PARTS.

Parts of a year.	*Parts of a month.*
6 months = $\frac{1}{2}$	15 days = $\frac{1}{2}$
4 " = $\frac{1}{3}$	10 " = $\frac{1}{3}$
3 " = $\frac{1}{4}$	6 " = $\frac{1}{5}$
2 " = $\frac{1}{6}$	5 " = $\frac{1}{6}$
$1\frac{1}{2}$ " = $\frac{1}{8}$	3 " = $\frac{1}{10}$
1 " = $\frac{1}{12}$	2 " = $\frac{1}{15}$

Note. When the rate per cent. per annum is - - $\left\{\begin{matrix}9\\8\\6\\4\\3\\2\end{matrix}\right\}$ multiply the principal by $\left\{\begin{matrix}\frac{3}{4}\\\frac{2}{3}\\\frac{1}{2}\\\frac{1}{3}\\\frac{1}{4}\\\frac{1}{6}\end{matrix}\right\}$ of the given number of months, and you will have the interest for the given period of time.*

EXAMPLES.

1. What is the interest of 347 dollars 50 cents, at 6 per cent. for one year?

$$\begin{array}{r}\$347{\cdot}50\\ 6\\ \hline 20{\cdot}8500\end{array}$$

Ans. $20 85c.

---

* The reason of this rule is, that as 9 is ¾ of 12 months, if the rate be 9 per cent. multiply the principal by ¾ of the months; if the rate be 8 per cent. by ⅔ of the number of months; and if by 6, by half the months, &c.

203. *What is the general rule for computing Simple Interest?*

2. What is the interest of $365 14cts. 6 mills, for three years, 7 months, and 6 days ?

365.146 principal.
6 rate.

6 months ½)21.90 876 interest for 1 year.
3

65.72 628 interest for 3 years.
1 month ⅙)10.95 438 interest for 6 months.
6 days ⅕) 1.82 573 interest for 1 month.
.36 514 interest for 6 days.

$78.87 153 interest for 3 years, 7 months, and 6 days; that is $78 87c. $1\frac{53}{100}$m.

NOTE.—Because 7 months are not an even part of a year, take two such numbers as are even parts, and which added together will make 7 (6 and 1)—6 months are ½ of a year, therefore for 6 months, divide the interest of one year by 2; again, 1 month is ⅙ of 6 months, therefore for 1 month, divide the interest of 6 months by 6.—For the days, because 6 days are ⅕ of a month, or of 30 days, therefore for 6 days, divide the interest of 1 month by 5. Lastly, add the interest of all the parts of the time together; the sum is the answer.

3. What is the interest of £329 17s. 6½d. for 3 years, 7 months, and 12 days, at 5 per cent. per annum ?

£	s.	d.			£	s.	d.	
329	17	6½	6 months.....½)	Then	16	9	10½	interest of 1 year.
		5					3	
16.49	7	8½			49	9	7½	do. of 3 years.
.20			1 month......⅙)		8	4	11¼	do. of 6 months.
9.87			14 days.........⅓)		1	7	5¾	do. of 1 month.
.12			2 days.........⅕)			9	1¾	do. of 10 days.
						1	9¾	do. of 2 days
10.52								
4					£59	13	0	*Answer.*
2.10								

4. What is the interest of $500 for 3 years, 9 months, and 25 days, at 6 per cent. ?

Ans. $114.583⅓.

5. What will be the interest of $60.625 for 2 years, 10 months, and 20 days, at 6 per cent. ?

Ans. $10.5083+.

6. What is the interest of $124 for 3 months and 10 days, at 6 per cent.? Ans. $2.066⅔.

7. What is the interest of $215.65 for 5 years, 5 months, and 6 days, at 6 per cent.?

Ans. $70.3019.

8. What will be the interest of 75 cents for 4 months, at 6 per cent.?

Ans. 1c. 5m.

To find the interest of any sum at 6 per cent. per annum, for any number of months.

Rule.—Multiply the principal by half the number of months, and divide the product by 100; the quotient will be the interest for the given time.*

EXAMPLES.

1. What is the interest of $225 for 4 months, at 6 per cent.?

```
$225 principal.
   2 half the number of months.
────
$4.50 Ans.
```

2. What is the interest of 1575 dollars for 10 months, at 6 per cent.?

To find the interest of any sum, for any number of days.

Rule.—Multiply the principal by the given number of days, and that product by the rate, then divide the last product by 365 × 100, and the quotient will be the interest.

---

* The reason of this rule. When the time is months, multiplying by the rate for the time, gives the answer. This rate is found by multiplying the time by the given rate per cent. for a year, and dividing the product by 12; the quotient is the rate required, and is always equal to half the number of months, when the yearly rate is 6 per cent.

204. *What is the rule for finding the interest of any sum at 6 per cent. per annum, for any number of months?*——205. *What is the reason of this rule?*

Rule 2. Multiply the principal by the given number of days, and divide the product by 6083 for 6 per cent. and 7300 for 5 per cent. (the days in which any sum will double at those rates,) and the quotient will be the interest.

EXAMPLES.

1. What will be the interest of 320 dollars for 146 days, at 6 per cent. ?

$ days. rate.
326×146×6=280320; and 365×100=36500.
Then 280320÷36500=$7·68 the interest.

2. What is the interest of $225·50 for 292 days, at 6 per cent. ?
Ans. $10·824.

3. What will be the interest of $225·50 for 292 days, at 6 per cent. ?
Ans. $10·324$\frac{3008}{6083}$.

4. What is the interest of 146 dollars for 146 days, at 5 per cent.?
Ans. $2·92.

To find the interest on bonds, notes of hand, &c. when partial payments have been made, or endorsed on them.

Rule 1. Find the interest of the given sum to the first payment, which either alone, or with any preceding payment, if any, exceeds the interest due at that time, and add that interest to the given sum.

Rule 2. From this amount subtract the payment made at that time with the preceding payments, if any, and the remainder will form a new principal; the interest of which find and subtract as of the first sum, and so on till the last payment.

Note.—This mode of computing interest, is established by law in Massachusetts for making up judgments on securities for money drawing interest, and on which partial payments are endorsed. This mode is the most equitable, because the payments are applied to keep down the interest, no part of which, in this method of computation, forms any part of the principal, drawing interest.

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206. *What are the rules for finding the interest of any sum for any number of days?* ——207. *What is the rule for computing interest upon bonds, notes, &c. upon which partial payments have been made?*

EXAMPLES.

1. *A.* gave a note to *B.* dated Jan. 1, 1820, for $1000, payable on demand, with interest; on which were the following endorsements:

			years.	*months.*
March 1, 1821,	received	75 dollars......	1	2
July 1, 1821,	"	20 dollars......		4
Sept. 1, 1822,	"	20 dollars......	1	4
Nov. 1, 1822,	"	750 dollars......		4
March 1, 1823,	"	100 dollars......		4

What was the balance due July 1, 1823, interest being computed at 6 per cent.?

Calculation.

	$	
	1000	Principal.
	70	Interest to March 1, 1821, 1 yr. 2 months.
	1070	Amount.
	75	First payment deducted.
	995	New principal March 1, 1821.
	19·90	Interest to July 1, 1821, 4 months.
	1014·90	Amount.
	20·00	Second payment deducted.
	994·90	New principal July 1, 1821.
	79·592	Interest to Nov. 1, 1822, 1 yr. 4 months.
	1074·492	Amount.
	20·000	Third payment Sept. 1, 1822.
	750·000	Fourth payment Nov. 1, 1822.
	770·000	Sum of 3d and 4th payments deducted.
	304·492	New principal Nov. 1, 1822.
	6·089	Interest to March 1, 1823, 4 months.
	310·581	Amount.
	100·000	Fifth payment deducted.
	210·581	New principal March 1, 1823.
	4·212	Interest to July 1, 1823, 4 months.
Ans.	$214·793	Balance due July 1, 1823.

☞ A simple method of operation, is, first to set down against each payment, (as in Ex. 1.) the time for which the interest is to be cast: then set down the sums, interest, payments, &c. in columns, as follows:

	Principal.	Time.	Interest.	Payments.	Excess.
	dolls. mills.	*months.*	*dolls. mills.*	*dolls. mills.*	*dolls. mills.*
1	1000·000	14	70·000	75·000	5·000
	5·000				
2	995·000	4	19·900	20·000	·100
	·100				
3	994·900	16	79·592	20·000	
4	" "	4	6·089	750·000	
			85·681	770·000	684·319
	684·319				
5	310·581	4	4·212	100·000	95·788
	95·788				
	$214.793 Balance due July 1, 1823.				

2. Supposing a note of 867 dollars 33 cents, dated January 6, 1814, upon which the following payments should be made, viz.

1. April 16, 1817, - - $136·44.
2. April 16, 1819, - - 319·
3. Jan. 1, 1820, - - 518·68,

What would be due July 11, 1821 ?

Ans. $215·103.

* * *

COMMISSION.

Commission is a premium, at so much per cent. allowed a person called a correspondent, factor, or broker, for assisting merchants, and others, in purchasing and selling goods.

Rule.—Multiply the given sum by the rate per cent., and cut off the two right hand figures, as in Simple Interest.

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208. *What is Commission ?*—209. *What is the rule for calculating commission ?*

EXAMPLES.

1. What is the commission on the purchase of goods, the invoice of which amounts to 1250 dollars, at 2½ per cent. ?

Ans. $31·25.

2. What must I allow my correspondent for selling goods to the amount of $4325·75 at a commission of 5 per cent. ?

Ans. $216·2875.

* * *

BUYING AND SELLING STOCKS.

Stock is a general name for the capitals of trading companies, or of a fund established by government, the value of which is variable.

Rule.—If the stock be above par, that is, when 100 dollars of stock are worth more than 100 dollars, multiply the given sum by its value per cent. over 100 per cent. and divide the product by 100; then add the quotient and given sum together, the amount will be the value required.

If the stock be below par, that is, when 100 dollars of stock are worth less than 100 dollars, multiply the given sum by its value per cent. less than 100 per cent. and divide the product by 100; then subtract the quotient from the given sum, the remainder will be the value required.

EXAMPLES.

1. What is the value of $7500 United States Bank Stock, at 112½ per cent. ?

Ans. $8437·50.

2. What is the value of $5400 of Stock, at 97 per cent. ?

Ans. $5238.

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210. *What are Stocks?*——211. *What is the rule for finding the value of any kind of stock, the value per cent. being given?*

INSURANCE.

Insurance is a security by contract, to indemnify for a specified sum, the insured for such loss or damage as may happen to the property, for a limited time.

The *premium* is the sum paid by the insured for the insurance of his property, and is generally at so much per cent.

Rule.—Multiply the sum to be insured by the rate per cent. and divide the product by 100;—the quotient will be the premium or answer.

EXAMPLE.

What will be the annual premium for insuring a house against loss by fire, valued at $5600, at ¾ per cent.? Ans. $42.

* * *

SIMPLE INTEREST BY DECIMALS.

A TABLE OF RATIOS FROM ONE DOLLAR, &C. TO TEN DOLLARS.

Ratio is the simple interest of $1 or £1 for one year, at the rate per cent. agreed on, and is found by dividing the rate by 100, and reducing it to a decimal. Thus $\frac{6}{100} = .06$, *and* $\frac{5}{100} = .05$.

Rate per cent.	ratios.	Rate per cent.	ratios.	Rate per cent.	ratios.
1	.01	4	.04	7	.07
1¼	.0125	4¼	.0425	7¼	.0725
1½	.015	4½	.045	7½	.075
1¾	.0175	4¾	.0475	7¾	.0775
2	.02	5	.05	8	.08
2¼	.0225	5¼	.0525	8¼	.0825
2½	.025	5½	.055	8½	.085
2¾	.0275	5¾	.0575	8¾	.0875
3	.03	6	.06	9	.09
3¼	.0325	6¼	.0625	9¼	.0925
3½	.035	6½	.065	9½	.095
3¾	.0375	6¾	.0675	9¾	.0975
				10	.1

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212. *What is Insurance?*——213 *What is the premium?*——214. *What is the rule for finding the premium, the rate per cent. being given?*

A TABLE for the ready finding of the decimal parts of a year, equal to any number of months and days, allowing 12 months, or 365 days to the year.

Days.	decimal parts.	Days.	Decimal parts.	Months.	decimal parts.
1	.00274	30	.082192	1	.083333
2	.005479	40	.109589	2	.166666
3	.008219	50	.136986	3	.25
4	.010959	60	.164383	4	.333333
5	.013699	70	.191781	5	.416666
6	.016438	80	.219178	6	.5
7	.019178	90	.246575	7	.583333
8	.021918	100	.273973	8	.666666
9	.024657	200	.547945	9	.75
10	.027397	300	.821918	10	.833333
20	.054794	365	1.000000	11	.916666

RULE.

1. Multiply the principal by the ratio, the product will be the interest for one year.

2. To find the interest for more years than one, multiply the interest of one year by the given number of years, and the product will be the interest for that number of years.

3. To find the interest for months, or months and days ; multiply the interest of one year by the decimal parts which are equal to the given number of months, or months and days, and the product will be the interest for that time.*

EXAMPLES.

1. What is the interest and amount of $475.50 for 4 years, 9 months, 25 days, at 6 per cent. ?

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* This rule is a contraction of the general rule for simple interest.

215. *What is the rule for computing simple interest by decimals ?*

```
                                  $  cts.
9 mo.=.75                        475 50 principal.
20 da.=.054794                       .06 Ratio.
5 da.=.013699                    ----------
      ---------                  28.5300 Int. for one year.
Years, 4.818493                  4.818493 Time.
                                 ----------
                                  855900
                                 2567700
                                1141200
                               2282400
                               285300
                              2282400
                             1141200
                             ---------------
              Interest.      137.4716052900
              Principal,     475.50
                             ----------
                             $612.9716 Amount, or answer.
```

2. If a factor sell a cargo for me, to the amount of $1750, on commission at 2¼ per cent, and purchase me another cargo of the value of $2500, on commission at 1¾ per cent; what will his commission on both cargoes amount to?

Ans. $83.125.

COMPOUND INTEREST.

Compound Interest is that which arises from the interest being added to the principal at the end of each year, and making the amount the principal for the next succeeding year.

Rule.—Find the interest of the given sum for one year, and add it to the principal, the amount is the principal for the second year; then find the interest of that amount, and add as before, and thus proceed for any number of years required.

---

216. *What is Compound Interest?*——217. *What is the rule for calculating compound interest?*

Subtract the first principal from the last amount, and the remainder will be the compound interest for the whole time.*

EXAMPLES.

1. What is the compound interest of $680 for 4 years, at 6 per cent?

$	
680	given sum or first principal.
40.80	interest of do. for one year.
720.80	amount or principal for 2nd year.
43.248	interest of do.
764.048	amount or principal for 3rd year.
45.843	interest of do.
809.891	amount or principal for 4th year.
48.593	interest of do.
858.484	amount for 4 years.
680.000	first principal subtracted.

Ans. $178.484 compound interest for 4 years.

2. What is the compound interest of £400, for 3 years, at 5 per cent.? Ans. £63 1s.

3. What will be the amount and compound interest of 1000 dollars for 5 years, at 6 per cent.?

Ans. { $1338.226 amount.
{ $ 338.226 comp. int.

[illegible]

---

* It is not usually necessary to carry the work beyond mills; therefore, when the figure next beyond mills, at the right is 5 or more than 5, increase the number of mills 1; when it is less than 5, it may be omitted, and the result will be exact enough for common purposes.

COMPOUND INTEREST BY DECIMALS.

TABLE showing the amount of $1, or £1, at 5 and 6 per cent. per annum, compound interest, from 1 year to 20 years.

Years.	5 per cent.	6 per cent.	Years.	5 per cent.	6 per cent.
1	1.050000	1.060000	11	1.710339	1.898298
2	1.102500	1.123600	12	1.795856	2.012196
3	1.157625	1.191016	13	1.885649	2.132928
4	1.215506	1.262479	14	1.979931	2.260903
5	1.276281	1.338225	15	2.078928	2.396558
6	1.340095	1.418519	16	2.182874	2.547271
7	1.407100	1.503630	17	2.292018	2.692772
8	1.477455	1.593848	18	2.406619	2.854339
9	1.551328	1.689478	19	2.526950	3.025599
10	1.628894	1.790847	20	2.653297	3.207135

RULE 1.—Multiply the given principal continually by the amount of one dollar, or one pound, for one year, at the rate per cent. given, until the number of multiplications be equal to the given number of years, the last product will be the amount for the whole time; from which subtract the given principal, and the remainder will be the compound interest. *Or,*

RULE 2.—Multiply the amount of one dollar, or one pound, for the given number of years, by the given principal, the product will be the amount required; from which subtract the given principal, and the remainder will be the compound interest.

EXAMPLES.

1. What will be the amount and compound interest of $500 for 3 years, at 6 per cent.?

By rule 1. $500 × 1.06 × 1.06 × 1.06 = $595.508 Amt.
$595.508—500 = 95.508 Comp. Int.

By rule 2. Amount of one dollar for 3 years = 1.191016
principal = 500

Amount $595.508000
500

Comp. Int. $95.508000

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219. *What is the method of computing compound interest by decimals?*

2. What will be the compound interest of £320 for 4 years, at 5 per cent.?

Ans. £68 19s. 2d. 3.52qrs. *Rule* 1.

3. What will $50 amount to in 20 years, at 6 per cent. compound interest?

Ans $160.35675. *Rule* 2.

4. What will be the compound interest of $1500 for 15 years, at 6 per cent.?

Ans. $2094.837. *Rule* 2.

◆

DUODECIMALS;*

OR

CROSS MULTIPLICATION.

The rule of Duodecimals is particularly useful to Workmen and Artificers, in casting up the contents of their work.

The denominations are feet, inches or primes, seconds, thirds, fourths, fifths, &c. there being no limit to the division.

12 fifths, (marked''''') are 1 fourth, ('''')
12 fourths - - - - - - - - - 1 third, (''')
12 thirds - - - - - - - - - 1 second ('')
12 seconds - - - - - - - - - 1 inch or prime, (')
12 inches or primes - - 1 foot (ft)

Glaziers' and Masons' work is measured by the foot.

Painting, plastering, and paving are done by the yard.

Partitioning, flooring, slating, rough boarding, by the square of 100 feet.

Stone and brick work by the rod of 16½ feet, whose square is 272¼. Bricks also are laid by the thousand.

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* *Duodecimals* are a species of compound numbers, decreasing in an uniform ratio of 12, from a greater denomination to a less; hence their name.

219 *What are duodecimals?*——220. *What is the use of duodecimals?*——221. *What are the denominations used in duodecimals?*

RULE.

1. Under the multiplicand write the corresponding denominations of the multiplier.

2. Multiply each term in the multiplicand, beginning at the lowest, by the feet in the multiplier, and write the result of each under its respective term, observing to carry an unit for every 12, from each lower denomination to its superior.

3. In the same manner multiply the multiplicand, by the inches in the multiplier, and write the result of each term in the multiplicand, thus multiplied, *one place* to the right hand in the product.

4. Proceed in the same manner with the other parts in the multiplier, which if seconds, write the result *two places* to the right hand; if thirds, *three places, &c.* and their sum will be the answer required.

Note.—Feet multiplied by feet give feet—Feet multiplied by inches give inches—Feet multiplied by seconds give seconds—Inches multiplied by inches give seconds—Inches multiplied by seconds give thirds—Seconds multiplied by seconds give fourths.

EXAMPLES.

Multiply 7 feet, 3 inches, 2 seconds, by 1 foot, 7 inches and 3 seconds.

	F.	I.	″		
	7	3	2		
	1	7	3		
Prod. by the feet,	7	3	2 ‴		
do. by primes,	4	2	10	2 ⁗	
do. by seconds,		1	9	9	6
	11	7	9	11	6

☞Here we multiply the 7f. 3in. 2″ by the 1f. in the multiplier, which gives seconds, inches, and feet. We next multiply the same 7f. 3in. 2″ by the 7in. saying 7 times 2 are 14, which is once 12 and 2 over, which (2) we set down one place to the right hand, that is, in the place of thirds, and carry one to the next place, and proceed in the same manner with the other terms. Lastly, we multiply the multiplicand by the 3″ saying 3 times 2 are 6, which we set down two places to the right hand and so proceed with the other terms of the multiplicand. The sum of all the products is the answer.

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222. *What is the rule?*

APPLICATION AND USE OF DUODECIMALS.

I. *To find the superficial contents of boards, &c. where length and breadth only are considered.*

Rule.—Multiply the length by the breadth, and the product will be the superficial content.

Note.—If the board or plank is tapering, add the width of both ends together, and take half the sum for the mean width, which multiplied by the length, will give the contents.

EXAMPLES.

1. How many feet in a board 10 feet 7 inches long, and 9 inches wide?

Ans. 7ft. 11′ 3″

2. What number of feet are there in a floor 16 feet 6 inches long, and 12 feet 8 inches wide?

Ans. 209 feet.

II. *To find the solid content of timber, stone, bales, trunks, &c.*

Rule.—Multiply the length by the breadth, and the product by the depth or thickness; the last product will be the content in solid or cubick feet, and parts of a foot.

EXAMPLES.

1. How many cubick feet in a stick of timber 12 feet 10 inches long, 1 foot 7 inches wide, and 1 foot 9 inches thick?

Ans. 35ft. 6.′ 8.″ 6.‴

2. How many cubick feet, and perches of $24\frac{3}{4}$ feet, are there in a cellar wall, 130 feet 8 inches long, 8 feet high, and 2 feet 9 inches thick?

Ans. 2874ft. 8.′ $=116\frac{44}{297}$ perches.

Note.—Bricklayers value their work at the rate of a brick and a half thick; and if the thickness of the wall is more or less, it must be reduced to that thickness, which may be done by the following

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223. What is the rule for measuring boards by duodecimals?——224. What is the rule for finding the solid content of timber, stone, &c.?

Rule.—Multiply the area of the wall by the number of the half bricks in the thickness of the wall, the product divided by 3 will give the area.

III. To measure drains, vaults, dikes, cellars, &c.

Rule.—Multiply the length, width, and depth in feet together, and divide by 216.

Note.—Diggers work by the square of 6 feet long, wide and deep, making 216 cubic feet to a square.

EXAMPLES.

1. There is a drain 200 feet long, 3½ feet wide, and 5¾ feet deep; how many squares does it contain?

Ans. 18$\frac{137}{216}$ squares.

2. How many squares are there in a vault 8 feet square, and 9½ feet deep?

Ans. 2$\frac{22}{27}$ squares.

IV. To measure wood.

Rule.—Multiply the length by the width, and the product by the height, the last product will be the content in cubick feet, and parts of a foot, which are brought into cords by dividing them by 128, or into cord wood feet by dividing by 16.

Note.—A cord of wood is a pile 8 feet long, 4 feet wide, and 4 feet high, containing 128 cubick feet, or 8 feet of cord wood. A foot of cord wood is a pile 4 feet long, 4 feet wide, and 1 foot high, containing 16 cubick feet.

EXAMPLES.

1. How many cords are there in a pile of wood 176 feet long, 3 feet 9 inches wide, and 4 feet 3 inches high?

Ans. 21$\frac{117}{128}$ cords.

---

225. What is the rule for measuring drains, vaults, &c.?—226. What is the rule for measuring wood duodecimally?

2. How many cord wood feet are there in a load of wood 8 feet long, 3 feet 10 inches wide, and 5 feet 4 inches high?

Ans. $10\frac{2}{9}$ feet.

3. How many cord wood feet are there in a pile 16 feet long, 5 feet 3 inches wide, and 4 feet 1 inch high?

Note 2.—As the superficial contents of one end of a cord of wood are exactly double to the number of cord wood feet, therefore, in order to find the number of feet of cord wood, in any load, multiply the height by the width, duodecimally, and divide the product by 2, the quotient will be the number of cord wood feet, and parts of a foot, which the load contains.

EXAMPLES.

1. How many feet of wood in a load 3ft. 6in. high, and 3ft. 6in. wide?

```
     F.  ′
     3   6
     3   6
    ------
    10   6
     1   9   0
    ----------
  2)12   3   0
    ----------
    6⅛ Ans.
```

2. How many feet of wood in a load 4ft. 3in. high, and 3ft. 5in. wide?

```
     F.  ′
     4   3
     3   5
    ------
    12   9
     1   9   3
    ----------
  2)14   6   3
    ----------
    7¼ Ans.
```

☞ In these examples, each load is considered as consisting of *two tiers*, each of which is supposed to be cut four feet long, according to law.

Note 3.—After having multiplied the height and width of any load of wood together, the figures which occupy the place of *inches*, in the product, are not *twelfth* parts of a foot, because, as they are to be divided by *two*, they are only *twenty-fourths* of a foot; therefore, 3 is $\frac{1}{8}$, 4 is $\frac{1}{6}$, 6 is $\frac{1}{4}$. 8 is $\frac{1}{3}$, and 9 is $\frac{3}{8}$. When the figures in the place of inches happen to be 5, 7, 10, or 11, as these figures are not *even* parts of 24, we call 5, $\frac{1}{6}$; 7, $\frac{1}{4}$; 10, $\frac{3}{8}$; 11, $\frac{3}{8}$; or $\frac{1}{2}$, as the case may be; that is, if the figure in the *third* place be less than 6, we call 11, $\frac{3}{8}$; but, if it be *more* than 6, then we call 11, $\frac{1}{2}$.

The figures in the *third* place are so inconsiderable, that they are not reckoned into the contents of a load.

3. How many feet of cord wood, in a load 3ft. 9in. high, and 2ft. 10in. wide ?

F. '
3 9
2 10

7 6
3 1 6

3)10 7 6

$5\frac{1}{4}$ Ans.

4 How many feet of cord wood, in a load 4ft. 7in. high, and 3ft. 8in. wide ?

F. '
4 7
3 8

13 9
3 0 8

2)16 9 8

$8\frac{3}{8}$ Ans.

In the above examples, the 7, which occupies the place of inches in the *one*, I call $\frac{1}{4}$ of a foot, although it is in reality *one twenty-fourth* part of a foot *more* than a quarter. The 9, which occupies the place of inches, in the other example, is exactly $\frac{3}{8}$ of a foot.

The 6, which possesses the *third* place, in one example, and the 8, in the other, are not reckoned into the quantity.

5. How many feet of cord wood in a load 4ft. 4in. high, and 3ft. 1in. wide ?

F. '
4 4
3 1

13 0
0 4 4

2)13 4 4

$6\frac{2}{3}$ Ans.

In this example 13 divided by 2 are $6\frac{1}{2}$, and 4, which occupies the place of inches, is $\frac{1}{6}$ of a foot; therefore $\frac{1}{2}$, which is $\frac{3}{6}$, being added to $\frac{1}{6}$, produces $\frac{4}{6}$, which is exactly $\frac{2}{3}$.

6. How many feet of cord wood are there in a load which is 3ft. 11in. high, and 3ft. 10in. wide ?

Ans. $7\frac{1}{2}$.

Note 4.—When wood is cut *less*, or *more*, than 4ft. long, find the contents of the load by the foregoing examples; then *deduct* or *add*, as the case may require, so many *forty-eighths* of a foot, as the number of feet in the load will produce when multiplied by the number of inches it falls short, or overruns.

7. How many feet of cord wood are there in a load 4ft. high, and 3ft. wide, and cut only 3ft. 9in. long ?

---

227. *When wood is cut more or less than 4 feet long, how do you find the contents of a load ?*

F. ′
4 0
3 0
2)12 0
6
$\frac{3}{8}$ deduct.
$5\frac{5}{8}$ Ans.

In this example, the contents of the load, in case it were cut 4ft. long, is 6ft. But as it lacks 3in. of 4ft. multiply 3 by 6, and the product is 18, which is *eighteen forty-eighths* ; and $\frac{18}{48} = \frac{3}{8}$, which being deducted from 6, leaves $5\frac{5}{8}$, the real quantity of the load.

8. How many feet of cord wood in a load, 4ft. 3in. high, and 3ft. 6in. wide, and cut 4ft. 7in. long ?

Ans. $8\frac{3}{8}$.

9. How many feet of cord wood in a load, 3ft. 7in. high, and 3ft. 8in. wide, and cut 3ft. 6in. long ?

Ans. $5\frac{3}{4}$.

10. How many feet of cord wood in a load, 4ft. 2in. high, and 3ft. 8in. wide ?

Ans. $7\frac{5}{8}$.

◆

THE SINGLE RULE OF THREE.

The Single Rule of Three teaches how to find a fourth number, proportional to three numbers given ; for which reason it is sometimes called the **Rule of Proportion**. It is called the Rule of Three, because *three* terms or numbers are given, to find a *fourth*. And because of its great and extensive usefulness, it is often called the **Golden Rule**.

This Rule is usually considered as of two kinds, viz. *Direct* and *Inverse* : a distinction, however, which is totally useless, and which has been avoided by some of the best writers. It may not be amiss, however, to explain the difference usually understood between the two parts of this rule.

The *Rule of Three Direct* is that in which more requires more, or less requires less. As in this ; if 3 men dig 21 yards of trench in a certain time, how much will 6 men dig in the same time ? Here more requires more, that is, 6 men, which are more than 3

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228. What are we to understand by the Single Rule of Three ?——229. Why is it called the Rule of Three, or the Golden Rule ?——230. Is it of more kinds than one ?

men, will also perform more work in the same time. Or, when it is thus: if 6 men dig 42 yards, how much will 3 men dig in the same time? Here less requires less, for 3 men will perform proportionably less work than 6 men, in the same time. In both these cases, then, the Rule, or the Proportion, is Direct: and the stating must be

thus, As 3 : 21 : : 6 : 42,
or thus, As 6 : 42 : : 3 : 21.

But the *Rule of Three Inverse*, is when more requires less, or less requires more. As in this: if 3 men dig a certain quantity of trench in 14 hours, in how many hours will 6 men dig the like quantity? Here it is evident that 6 men, being more than 3, will perform an equal quantity of work in less time or fewer hours.—Or thus: if 6 men perform a certain piece of work in 7 hours, in how many hours will 3 men perform the same? Here less requires more, for 3 men will take more hours than 6 to perform the same work. In both these cases then the Rule, or the Proportion, is Inverse; and the stating must be

thus, As 6 : 14 : : 3 : 7,
or thus, As 3 : 7 : : 6 : 14.

And in all these statings, the fourth term is found, by multiplying the 2d and 3d terms together, and dividing the product by the 1st term.

Of the three given numbers, two of them contain the supposition, and the third a demand. And for stating and working questions of these kinds, observe the following

GENERAL RULE.

1. Write that number which is of the same name or kind with the answer or number required, for the *second* term.

2. Then consider whether the answer must be greater or less than the second term. If the answer must be greater than the second term, write the greater of the two remaining numbers on the right for the *third* term, and the other on the left for the *first* term;—but if the answer must be less than the second term, write the less of the two remaining numbers on the right for the *third* term, and the other on the left for the *first* term.

3. Multiply the second and third terms together, divide by the first, and the quotient will be the answer to the question, which,

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231. *What is the nature of these useless distinctions?*—282. *What is the general rule for stating and performing all questions in simple proportion?*

(as also the remainder) will be in the same denomination in which you left the second term, and may be brought into any other denomination required.*

NOTE.--The chief difficulty that occurs in the *Rule of Three*, is the right placing of the numbers, or stating of the question; this being accomplished, there is nothing to do, but to multiply and divide, and the work is done.

To this end the nature of every question must be considered, and the circumstances on which the proportion depends, observed, and common sense will direct this if the terms of the question be understood.

1. The fourth number is always found in the same name in which the second is given, or reduced to; which, if it be not the highest denomination of its kind, reduce to the highest, when it can be done.

2. When the second number is of divers denominations, bring it to the lowest mentioned, and the fourth will be found in the same name

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* This rule is founded on the obvious principle, that the magnitude or quantity of any effect varies constantly in proportion to the varying part of the cause: Thus, the quantity of goods bought, is in proportion to the money laid out;—the space gone over by an uniform motion. is in proportion to the time, &c.

It has been shewn in Multiplication of Money, that the price of one multiplied by the quantity, is the price of the whole; and in Division, that the price of the whole divided by the quantity, is the price of one Now, in all cases of valuing goods, &c. where one is the first term of the proportion, it is plain, that the answer found by this rule, will be the same as that found by Multiplication of Money; and where one is the last term of the proportion, it will be the same as that found by Division of Money.

In like manner, if the first term be any number whatever, it is plain that the product of the second and third terms will be greater than the true answer required, by as much as the price in the second term exceeds the price of one, or as the first term exceeds an unit; consequently this product, divided by the first term, will give the true answer required.

NOTE 1.—When it can be done, multiply and divide as in Compound Multiplication, and Compound Division.

2. If the first term, and either the second or third can be divided by any number without a remainder. let them be divided, and the quotient used instead of them.

☞ *The following methods of operation, when they can be used, perform the work in a much shorter manner than the general rule.*

1. Divide the second term by the first; multiply the quotient into the third, and the product will be the answer.

2. Divide the third term by the first; multiply the quotient into the second, and the product will be the answer.

3. Divide the first term by the second, and the third by that quotient, and the last quotient will be the answer.

4. Divide the first term by the third, and the second by that quotient, and the last quotient will be the answer.

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233. *What is the reason of this rule; or why will multiplying the second and third terms together, &c. produce the true answer?*

to which the second is reduced, which reduce back to the highest possible.

3. If the first and third be of different names, or one or both of divers denominations, reduce them both to the lowest denomination mentioned in either.

4. When the product of the second and third is divided by the first; if there be a remainder, after the division, and the quotient be not the least denomination of its kind; then multiply the remainder by that number, which one of the same denomination with the quotient contains of the next less, and divide this product again by the first number; and thus proceed till the least denomination be found, or till nothing remain.

5. If the first number be greater than the product of the second and third; then bring the second to a lower denomination.

6. When any number of barrels, bales, packages or pieces are given, each containing an equal quantity, let the contents of one be reduced to the lowest name, and then multiplied by the given number of packages or pieces.

7. If the given barrels, bales, pieces, &c. be of unequal contents, (as it most generally happens) put the separate content of each properly under one another, then add them together and you will have the whole quantity.

8. When any of the terms are given in *Federal Money*, the operation is conducted in all respects as in simple numbers, observing only to place the point or separatrix between dollars and cents, to point off the results according to what has been taught already in *Federal Money*, and *Decimal Fractions*.

Proof.—The method of proof is by inverting the question. That is, put the fourth number, or answer, in the first place; the third in the second; and the second in the third; work as before directed, and the quotient will be the first number.

EXAMPLES.

1. If 4 yards of cloth cost 16 dollars, what will 12 yards come to, at the same rate?

yds. $ yds.
4 : 16 :: 12
12
——
4)192
———
$48 Ans.

☞ In this question the answer must be money, therefore we write 16 dollars for the second term. As 12 yards must cost more than 4 yards, we write 12 yards on the right of 16 dollars for the third term, and 4 yards on the left for the first term.

---

234. When the first, second, or third terms are of different denominations, how do you proceed?——235. What is the method of proof?

2. *Proof of Example* 1.—If 48 dollars will pay for 12 yards of cloth, how many yards can be bought for 16 dollars ?

$ yds. $
48 : 12 : : 16
12
—
48)192(4 yds. Ans.
192
—

☞ In this question the answer must be yards; therefore, we write 12 yards for the second term. As 16 dollars will pay for a less number of yards than 48 dollars, we write 16 dollars on the right of the 12 yards for the third term, and 48 dollars on the left for the first term.

3. If 5 yds. of cloth cost $10, what will 20 yds. come to?

yds. $ yds.
As 5 : 10 : : 20
10÷5=2
—
$40 Ans.

Here we divide the 2d term by the 1st, and multiply the quotient into the 3d, for the answer.

yds. $ yds.
As 5 : 10 : : 20
4=20÷5
—
$40 Ans.

Here we divide the 3d term by the 1st, and multiply the quotient into the 2d, for the answer.

4. If 20 yds. cost $120, how many yards may be bought for $30 ?

$ yds. $
As 120 : 20 : : 30
120÷20=6 quot. and 30÷6=5 yards, Answer.

Here we divide the 1st term by the 2d, and then, the 3d term by the quotient for the answer.

$ yds. $
Again, As 120 : 20 : : 30
120÷30=4 quot. and, 20÷4=5 yards, Ans.

Here we divide the 1st term by the 3d, and then, the 2d term by that quotient for the answer.

5. If 1 cwt. of tobacco cost £5 12s. 9½d. what will 8 cwt. do. cost?

Cwt. £ s. d. Cwt.
As 1 : 5 12 9½ : : 8
8

Ans. £45 2 4

Here there is no need of reducing the middle term, because it can be performed by compound multiplication, the first term being an unit.

6. If 8 cwt. of tobacco cost £45 2s. 4d.; what is that per cwt?

£ s. d.
8)45 2 4

Ans. 5 12 9½

Here there is no need of reducing the middle term, because it may be performed by compound division only, the 3d term being an unit.

7. If 2 cwt. 1 qr. 14 lbs. of sugar be worth $21.75, what will be the value of 42 cwt. 3 qrs. at the same rate?

Cwt. qr. lb. $ c. Cwt. qrs.
2 1 14 : 21.75 : : 42 3
4 4

9 qrs. 171 qrs.
28 28

76 1368
19 342

266 lbs. 4788 lbs.
2175

23940
33516
4788
9576
$ c.
266)10413900(391.50 A.
798

2433
2394

399
266

1330
1330

☞ In this example, the first and third terms are compound numbers, therefore, we reduce them both to the least denomination mentioned in either, viz. pounds. The second term is reduced to the least denomination mentioned in it, by calling it all cents.—We then multiply the third term by the second, and divide the product by the first term, and the quotient is 39150 cents, which we reduce to dollars by pointing off the two right hand figures.

8. If 9 cwt. 3 qrs. of sugar cost £27 17s. 6d., what will 2 cwt. 1qr. 1 lb. cost?

Cwt. qrs.	£ s. d.	Cwt. qr. lb.
9 3	27 17 6	2 1 1
4	20	4
39	557	9
28	12	28
312	6690	73
78		19
1092		263

lb. d. lb.
As 1092 : 6690 : : 263
263
2007
4014
1338
12
1092)1759470(1611
1092 2|0)13|4 3d.
6674
6552 £6 14s. 3d. Answer.
1227
1092
1350
1092
258
4
1092)1032(0 qr.

☞ Here it will be seen, that the first and third terms are of the same kind, but of different denominations, and therefore are reduced to the same name or denomination, and that the demand of the question lies on the 3d term.

2. That the middle term, being given in pounds, shillings and pence, is reduced to pence. But

3. If the second term were in federal money it would be sufficient to proceed according to decimals.

9. If my income be 109 guineas per annum, I desire to know what I may spend per day, so that I may lay up £45 at the year's end?

Ans. £0 5s. 10¾d. $\frac{1}{365}$ per day.

NOTE 1.—You must subtract £45 from the value of 109 guineas.

2. There being 365 days in a year, your question must next be stated thus:

D. *Guin.* *£* *D.* *s.* *d,* *qrs.*
As 365 : 109—45 : : 1 : 5 10 3$\frac{1}{365}$ the Ans.

10. If my salary be £43 12s. 5d. per annum, what does it amount to per week?

Ans. £0 16s. 9$\frac{17}{52}$d.

The Stating.
W. £ s. d. W.
As 52 : 43 12 5 : : 1 : the Ans.

NOTE.—As there are 52 weeks and 1 day in a year, you will get the true answer to the above question by the following ratio.

D. *£* *s.* *d.* *D.*
As 365 : 43 12 5 : : 7 : 16s. 8$\frac{283}{365}$d.

11. Suppose my income to be 16s. 8$\frac{283}{365}$d. per week, what is it per annum.

Ans. £43 13s. 7¼ $\frac{51}{365}$d.

NOTE 1.—You must first reduce the middle term to pence.
2. You must multiply by 365 (the denominator of the fraction) and add to the product the 283 which remains; and remember always to do so in similar cases.
3. You must divide by 7, the first term, and the quotient will be the answer in 365ths of a penny, which (in all similar cases) must be first divided by the denominator, and then brought into pounds.

12. How many yards of cloth may be bought for $195 75c. of which 9½ yds. cost $11 2c.?

Ans. 168 yds. 3 qrs.

13. A merchant, failing in trade, owes in all £3475, and has in money and effects but £2316 13s. 4d.: Now, supposing his effects are delivered up, pray, what will each creditor receive on the pound?

£ £ s. d. £
As 3475 : 2316 13 4 : : 1 : £0 13s. 4d. Ans.

14. If the distance from Newburyport to York be 31 miles; I demand how many times a wheel, whose circumference is 15½ feet, will turn round in performing the journey?

Ans. 10560 times.

15. What will 37½ gross of buttons come to at 13 cents per dozen?

Ans. $58 50c.

16. If $\frac{7}{16}$ of a ship cost $1163 what is the whole worth?

Ans. $2658 28c. 5m.

17. A merchant bought 9 packages of cloth, at 3 guineas for 7 yards; each package contained 8 parcels, each parcel 12 pieces, and each piece 20 yards; how many dollars came the whole to, and how many per yard?

yds. guin. pack. $
As 7 : 3 : : 9 : 34560 Ans. for the whole cost.

yds. guin. yd. $
As 7 : 3 : : 1 : 2 Ans. per yard.

18. A merchant bought 49 tuns of wine for $910: freight cost $90; duties $40; cellar $31 67c.; other charges $50, and he would gain $185 by the bargain; what must I give him for 23 tuns?

Tuns. $ $ $ $ c. $ $ Tuns. $
As 49 : 910+90+40+31 67+50+185 : : 23 : 613 33c. Ans.

19. The earth being 360 degrees in circumference, turns round on its axis in 24 hours; how far does it turn in one minute, in the 43d parallel of latitude; the degree of longitude, in this latitude, being about 51 statute miles?

H. D. M. M. M.
As 24 : 360 × 51 : : 1 : 12¾ Ans.

20. If a staff, 4 feet long, cast a shade (on level ground) 7 feet, what is the height of that steeple, whose shade, at the same time measures 198 feet?*

F. sh. F. hei. F. sh. F. hei.
As 7 : 4 : : 198 : 113⅐ Ans.

---

* As the rays of light from the sun may be considered parallel, the *length* of the shadows must be proportioned to the heights of the objects. Hence the reason of the statement of this question.

21. Said Harry to Dick, my purse and money are worth $3\frac{1}{4}$ guineas, but the money is worth eleven times as much as the purse; pray how much money is there in it?

Ans. £4 3s. 5d.

22. If $\frac{3}{8}$ of a yard cost $\frac{4}{5}$ of a £, what will $\frac{7}{11}$ of a yard cost?*

As $\frac{3}{8} : \frac{4}{5} :: \frac{7}{11} : \frac{4}{5} \times \frac{7}{11} \div \frac{3}{8} = \frac{224}{165}$£ Answer.

Or, $\frac{8}{3} : \frac{4}{5} :: \frac{7}{11} : \frac{8}{3} \times \frac{4}{5} \times \frac{7}{11} = \frac{224}{165}$£$=$£1 7s. $1\frac{27}{33}$d.

23 There is a cistern, having four cocks; the first will empty it in ten minutes; the second in 25 minutes; the third in 40, and the fourth in 80 minutes; in what time will all four, running together empty it?

$$\text{As}\left\{\begin{matrix}10\\20\\40\\80\end{matrix}\right\}\begin{matrix}\text{Cist.} & & \text{Min.}\\ :1 & :: & 60 :\end{matrix}\quad \left\{\begin{matrix}6\\3\\1\frac{1}{2}\\\frac{3}{4}\end{matrix}\right\}\quad \begin{matrix} & \text{Cist.} & & \text{Min.} & & \text{Cist.} & & \text{Min.} \\ \text{As} & 11\frac{1}{4} & : & 60 & :: & 1 & : & 5\frac{1}{3}\ \text{Ans.} \\ \text{that is} & \frac{45}{4} & : & 60 & :: & 1 & : & \frac{60\times4}{45}=5\frac{1}{3}.\end{matrix}$$

$$\overline{11\frac{1}{4}}\ \text{Cist.}$$

24. If the earth revolves 366 times in 365 days, in what time does it perform one revolution?

Ans. 23h. 56′ 3″ 56‴+=1 Sidereal day.†

25. If the earth makes one complete revolution in 23h. 56′ 3″+, in what time does it pass through one degree?

Ans. 3′ 55″ 20‴.

26. If the earth performs its diurnal revolution in a solar day,‡ or 24 hours; in what time does it move one degree?

Ans. 4′.

---

* If the first term of the statement be a Vulgar Fraction, whether the other terms are or not, after the first and third terms are reduced to the same denomination, invert the first term as in division of Vulgar Fractions, and the product of the three terms will of course be the answer.

The student should work the questions in Vulgar, or Decimal Fractions, according as the rules for fractions require.

† A sidereal day is the space of time which happens between the departure of a star from, and its return to the same meridian again.

‡ The solar day is that space of time which intervenes between the sun's departing from any one meridian, and its return to the same again.

27. If $\frac{3}{5}$ yd. cost $\$\frac{7}{8}$, what will $40\frac{1}{2}$ yds. come to?

Ans. \$59 6c. $2\frac{1}{2}$m.

28. At $\$3\frac{5}{8}$ per cwt. what will $9\frac{2}{3}$ lb. come to?

Ans. 31c. 3m.

29. A conduit has a cock, which will fill a cistern in ·2 of an hour; this cistern has 3 cocks; the first will empty it in 1·25 hour, the second in ·625 of an hour, and the third in ·5 hour. In what time will the cistern be filled, if all four run together?

Ans. 1h. 40m.

30. In a certain school, $\frac{1}{20}$th of the pupils study Greek, $\frac{1}{10}$ study Latin, $\frac{2}{5}$ study Arithmetick, $\frac{1}{4}$ read and write, and 20 attend to other things; what is the number of pupils?

$\frac{1}{20}+\frac{1}{10}+\frac{2}{5}+\frac{1}{4}=\frac{16}{20}$, then $20=\frac{4}{20}$ and $\frac{4}{20} : 20 :: \frac{20}{20} : 100$ Ans.

* * *

GENERAL METHOD OF MAKING TAXES.

Rule.—In the first place an inventory of the value of all the estates, both real and personal, and the number of polls for which each person is rateable, must be taken in separate columns: the most concise way is then to make the total value of the inventory the first term, the tax to be assessed the second, and \$1 the third, and the quotient will show the value on the dollar: 2dly, make a table, by multiplying the value on the dollar by 1, 2, 3, 4, &c.—3dly, From the inventory take the real and personal estates of each man, and find them separately in the table, which will shew you each man's proportional share of the tax for real and personal estates.

Note. If any part of the tax is averaged on the polls, or otherwise, before stating, to find the value on the dollar, you must deduct the sum of the average tax from the whole sum to be assessed; for which average you must have a separate column, as well as for the real and personal estates.

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236. What method would you pursue were you to undertake the assessing of town taxes?

EXAMPLES.

1. Suppose the General Court should grant a tax of $500000, of which the town of Portsmouth is to pay $5312 50c. and, of which the polls, being 1550, are to pay $1 25c. each ;—The town's inventory amounts to $450000, what will it be on the dollar, and what is A's tax, whose estate (as by the inventory) is as follows, viz. real $1376, personal $1149, and he has 3 polls ?

Pol. $ c. Pol. $ c.
First, As 1 : 1 25 :: 1550 : 1937 50 the average part of the tax to be deducted from $5312 50c. and there will remain $3375.

$ $ $
Secondly, As 450000 : 3375 : : 1 : 7½m. on the dollar.

TABLE.

$	$	c.	m.	$	$	c.	m.	$	$	c.
1 is	0	0	7½	20 is	0	15	0	200 is	1	50
2—	0	1	5	30—	0	22	5	300—	2	25
3—	0	2	2½	40—	0	30	0	400—	3	00
4—	0	3	0	50—	0	37	5	500—	3	75
5—	0	3	7½	60—	0	45	0	600—	4	50
6—	0	4	5	70—	0	52	5	700—	5	25
7—	0	5	2½	80—	0	60	0	800—	6	00
8—	0	6	0	90—	0	67	5	900—	6	75
9—	0	6	7½	100—	0	75		1000—	7	50
10—	0	7	5							

Now to find what A's rate will be,
His real estate being $1376, we find by this table, that $1000 is - - - $7 50c.
that $300 is - 2 25
that $70 is - - 52 5m.
and that $6 is - 4 5

For his real estate $10 32

Real.	Personal.	Polls.	Total.
$ c. m.	$ c. m.	$ c. m.	$ c. m.
10 32 0	8 61 7½	3 75	22 68 7½

In like manner we find his tax for personal estate to be $8 61 7½ } +$10 32=$22 68c.7½m.
His 3 polls, at $1 25c. each are 3 75

or, $22 69c. Ans.

2. Suppose a tax of $765 be laid on a town, and the inventory of all the estates in town amounts to $9345, what must A. pay, whose estate is $149?

Ans. $12 038 nearly.

RULE OF THREE DIRECT.

The Rule of Three Direct teaches, by having three numbers given, to find a fourth, which shall have the same ratio to the second, as the third has to the first.

RULE.

1. State the question by making that number, which asks the question,* the third term; that which is of the same name or kind, the first term; and that which is of the same name or kind with the answer, the second term.

2. Multiply the second and third terms together, and divide the product by the first term, and the quotient will be the fourth term, or answer.

The notes under the general rule are applicable to this rule.

EXAMPLES.

1. If 3 bushels of corn be worth $1·80, what is the value of 12 bushels?

bu. $ c. bu.
3 : 1·80 : : 12
12

3)2160

$7·20 Ans.

In this example, 12 bushels asks the question, and is made the third term; 3 bushels being of the same name, is made the first term; and $1.80 being of the same name with the answer, is made the second term. Here the third term is greater than the first, and the question evidently requires the fourth term or answer to be greater than the second; therefore, the question belongs to the Rule of Three Direct.

2. If 6 lbs. sugar cost 10s. what will 33 lbs. cost at the same rate?
Ans. £2 15s.

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* The term which asks or moves the question has generally some words like these before it, viz. What will? What cost? How many? How far? How much? How long? &c.

237. *What is the method of operation in the Rule of Three Direct.*

RULE OF THREE INVERSE;

OR,

RECIPROCAL PROPORTION.

The Rule of Three Inverse teaches, by having three numbers given, to find a fourth, which shall have the same ratio to the second, as the first has to the third.

Rule.—State and reduce the terms as in the Rule of Three Direct; then multiply the first and second terms together, and divide the product by the third, the quotient will be the fourth term, or answer.

EXAMPLES.

1. If 6 men can do a piece of work in 18 days, in what time can 12 men do it?

m. d. m.
6 : 18 : : 12
6
——
12)108
——
9 days Ans.

In this example, the third term is greater than the first, yet it is evident, that the question requires the fourth term or answer to be less than the second; therefore, this question belongs to the Rule of Three Inverse.

2. If a man perform a journey in 15 days, when the day is 12 hours long, in how many days will he do it, when the day is but 10 hours?

Ans. 18 days.

THE DOUBLE RULE OF THREE.

The Double Rule of Three, or Compound Proportion, is the method of resolving such questions as require two or more operations by Simple Proportion.

It always consists of an odd number of terms given, as five, seven, &c. These terms are distinguished into *terms of supposition*, and *terms of demand*; the number of the former always exceeding that of the latter by one, which is of the same name or kind with the answer or term required.

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238. *How do you proceed in the Rule of Three Inverse?*——239 *What is Compound Proportion, or the Double Rule of Three?*——240. *How is it distinguished from Simple Proportion?*

RULE.

1. State the question, by placing the three conditional terms in such order that that number which is the cause of gain, loss, or action, may possess the first place; that which denotes space of time, or distance of place, the second; and that which is the gain, loss, or action, the third

2. Place the other two terms, which move the question, under those of the same name.

3. Then, if the blank place, or term sought, fall under the third place, the proportion is direct, therefore, multiply the three last terms together, for a dividend, and the other two for a divisor; then the quotient will be the answer.

4. But if the blank fall under the first or second place, the proportion is inverse, wherefore multiply the first, second and last terms together, for a dividend, and the other two, for a divisor; the quotient will be the answer.*

EXAMPLES.

1. If a principal of $100 gain $6 interest in one year, what will $500 gain in 8 months?

Statement & Operation.

```
$P.   M.   $INT.
100 : 12 : : 6
500 :  8
--------------
100          6
 12          8
---         --
1200        48
           500
          ----
   12|00)240|00(
          ---
        $20 Ans.
```

In this question the cause of $6 gain is $100; therefore, $100 possesses the first place in the statement. The time, 12 months, is set in the second place, as the rule directs; and the gain, $6, in the third place. Then, the other two terms being placed under those of the same name, the blank falls under the third place: therefore the proportion is *direct*; consequently, I multiply the three last terms, 6, 8, and 500, together, and take the product, 24000, for the dividend; and the product of the two first terms, 100 and 12 = 1200, is the divisor. The dividend being divided by the divisor, the quotient, $20, is the Ans. sought.

2. If $100 in 12 months will gain $6 interest; in what time will $750 gain $30 interest? Ans. 8 months.

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* When a question contains more than *five given terms* it can always be solved by several statements in the *Single Rule of Three*, and so may any question in the Double Rule.

241. *Repeat the rule?*

3. If 7 men can reap 84 acres of wheat in 24 days; how many men can reap 100 acres in 10 days?

Ans. 20 men.

4. If 12 men can earn £8 8s. in 8 days; what will 21 men earn in 15 days?

Ans. £27 11s. 3d.

5. If a family of 9 persons spend 450 dollars in 5 months; how much would be sufficient to maintain them 8 months, if 5 persons more were added to the family?

Ans. 1120 dolls.

6. If 2 men in 6 days of 12 hours each, build 30 rods of wall, how many hours long is the day when 8 men build 64 rods in 4 days?

Remark.—In this question as there are seven given terms or numbers, it cann[illegible] be solved by the rule given, but must be wrought by statements in the Single Rule of Three as observed in the note.

1. *By separate statements.*

M. H. M. H.
As 8 : 12 : : 2 : 3
D. H. D. H.
As 4 : 3 : : 6 : 4½
R. H. R. H.
As 30 : 4½ : : 64 : 9·6 Ans.

Explanation.—In the 1st statement, as more men will require less time, the 3rd term must be *less* than the 1st. In the 2d statement, less days require more hours, therefore, the 3rd term must be *greater* than the 1st. In the 3rd statement, more rods require more men, therefore the 3rd term must be greater than the 1st; observing, always, that the 4th term or answer resulting from any statement, must be the 2d term in the *next* statement.

2. *By reducing all the statements to one.*

$$\left.\begin{array}{r}8\text{m.}\\4\text{d.}\\30\text{r.}\end{array}\right\}\begin{array}{c}\text{h.}\\12\\ \end{array}\left\{\begin{array}{l}2\text{m.}\\6\text{d.}\\64\text{r.}\end{array}\right.$$

960 : 12 : : 768 : 9·6h. as before.

Explanation.—The 1st and 3d terms in the first three statements are the same as those in the separate statements. The middle term, by this method, must always be of the same name as the answer required. All the given first terms being multiplied together, the product will be the 1st term of the last statement, and the product of all the third terms will be the 3d term. Then the product of the 2d and 3rd terms divided by the 1st, will give the answer.

CONJOINED PROPORTION.

Conjoined Proportion is when the coins, weights or measures of several countries are compared in the same question; or, in other words, it is joining many proportions together, and by the relation which several antecedents have to their consequents, the proportion between the first antecedent and last consequent is discovered, as well as the proportion between the others in their several respects.

Note.—This rule may generally be abridged by cancelling equal quantities or numbers that happen to be the same in both columns; and it may be proved by as many statings in the Single Rule of Three as the nature of the question may require.

CASE I.

When it is required to find how many of the first sort of coin, weight or measure, mentioned in the question, are equal to a given quantity of the last.

Rule.—Place the numbers alternately, that is, the antecedents at the left hand and the consequents at the right, and write the last number on the left hand; then multiply all the numbers in the left hand column continually together for a dividend; and all the numbers in the right hand column for a divisor; divide, and the quotient will be the answer.

EXAMPLES.

1. If 12 lbs. of Boston be equal to 10 lbs. of Amsterdam, and 10 lbs. of Amsterdam be equal to 12 lbs. of Paris; how many pounds of Boston are equal to 80 lbs. of Paris?

Antecedents.	Consequents.	
12 of Boston	$=10$ of Amster.	Then $12\times10\times80=9600$ dividend; and
10 of Amster.	$=12$ of Paris,	$10\times12=120$ the divisor;
80 of Paris ?		Therefore $9600\div120=80$ lbs. Answer.

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242. What is the meaning of Conjoined Proportion?——243. When you wish to know how many of the first sort of coin, &c. mentioned in any question are equal to a given quantity in the last, how do you proceed?

2. If 140 braces of Venice be equal to 150 braces of Leghorn, and 7 braces of Leghorn be equal to 4 American yards; how many Venitian braces are equal to 32 American yards?

Ans. $52\frac{4}{15}$ braces.

CASE II.

When it is required to find how many of the last sort of coin, weight, or measure, mentioned in the question, are equal to a given quantity of the first.

Rule.—Place the numbers alternately, beginning at the left hand, and write the last number on the right hand; then multiply all the numbers in the right hand column continually together for a dividend, and all the numbers in the left hand column for a divisor; divide, and the quotient will be the answer.

EXAMPLES.

1. If 12 lbs. of Boston be equal to 10 lbs. of Amsterdam, and 100 lbs. of Amsterdam be equal to 120 lbs. of Paris; how many pounds of Paris are equal to 80 lbs. of Boston?

Antecedents.	Consequents.	
12 of Boston	=10 of Amsterdam.	$10\times120\times80=96000$ dividend;
100 of Amsterdam	=120 of Paris.	and $12\times100=1200$ divisor;
	80 of Boston?	Then $96000\div1200=80$ lbs. Ans.

2. If 140 braces of Venice be equal to 150 braces of Leghorn, and 7 braces of Leghorn be equal to 4 American yards; how many American yards are equal to $52\frac{4}{15}$ Venetian braces?

Ans. 32 yards.

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244. *How, when you wish to ascertain how many of the last are equal to a certain quantity named in the first?*

PRACTICE.*

Practice is a contraction of the Rule of Three Direct, when the first term happens to be an unit or one; it has its name from its daily use among Merchants and tradesmen, being an easy and concise method of working most questions which occur in trade and business.

Proof. By the Single Rule of Three, Compound Multiplication, or by varying the parts.

Before any advances are made in this rule, the learner should commit to memory the following

TABLES OF ALIQUOT, OR EVEN PARTS.

Aliquot, or even parts of Money.

Pts. of a shill. of a £.		
d.	s.	£
6	$\frac{1}{2}$	$\frac{1}{40}$
4	$\frac{1}{3}$	$\frac{1}{60}$
3	$\frac{1}{4}$	$\frac{1}{80}$
2	$\frac{1}{6}$	$\frac{1}{120}$
1½	$\frac{1}{8}$	$\frac{1}{160}$
1	$\frac{1}{12}$	$\frac{1}{240}$
¾	$\frac{1}{16}$	$\frac{1}{320}$
½	$\frac{1}{24}$	$\frac{1}{480}$
¼	$\frac{1}{48}$	$\frac{1}{960}$

Parts of 2 Shill.	
d.	2s.
1	$\frac{1}{24}$
1½	$\frac{1}{16}$
2	$\frac{1}{12}$
3	$\frac{1}{8}$
4	$\frac{1}{6}$
6	$\frac{1}{4}$
8	$\frac{1}{3}$

Parts of a Pound.		
s.	d.	£.
10	0	$\frac{1}{2}$
6	8	$\frac{1}{3}$
5	0	$\frac{1}{4}$
4	0	$\frac{1}{5}$
3	4	$\frac{1}{6}$
2	6	$\frac{1}{8}$
1	8	$\frac{1}{12}$
1	4	$\frac{1}{15}$
1	3	$\frac{1}{16}$
1	0	$\frac{1}{20}$
0	10	$\frac{1}{24}$
0	8	$\frac{1}{30}$
0	5	$\frac{1}{48}$
0	4	$\frac{1}{60}$
0	3	$\frac{1}{80}$
0	2	$\frac{1}{120}$

Parts of a Dollar.	
c.	$
50	$\frac{1}{2}$
33⅓	$\frac{1}{3}$
25	$\frac{1}{4}$
20	$\frac{1}{5}$
16⅔	$\frac{1}{6}$
12½	$\frac{1}{8}$
8⅓	$\frac{1}{12}$
6¼	$\frac{1}{16}$
5	$\frac{1}{20}$
4	$\frac{1}{25}$
3⅓	$\frac{1}{30}$
2	$\frac{1}{50}$
1	$\frac{1}{100}$

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* Perhaps no method can be more simple and concise to find the value of goods in Federal Money, than the general rule of multiplying the price by the quantity, as given in Multiplication of Federal Money or Decimals; therefore, the application of this rule to Federal Money is almost useless. Yet, as English merchants, trading with Americans, make out the invoices of their goods in sterling money, an acquaintance with this excellent rule is necessary to every one, employed in mercantile pursuits.

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245 *What is Practice?*—246. *Explain to me the use of the tables?*—247. *What are aliquot parts of any quantity?*

Aliquot, or even parts of Weight.

Parts of a Cwt.			Parts of $\frac{1}{2}$ Cwt.		Parts of $\frac{1}{4}$ Cwt.		Parts of a Ton.		
Qrs.	lb.	Cwt.	lb.	$\frac{1}{2}$ Cwt.	lb.	$\frac{1}{4}$ Cwt.	Cwt.	qr.	T.
2	0	= $\frac{1}{2}$	28	= $\frac{1}{2}$	14	= $\frac{1}{2}$	10	0	= $\frac{1}{2}$
1	0	= $\frac{1}{4}$	14	= $\frac{1}{4}$	7	= $\frac{1}{4}$	5	0	= $\frac{1}{4}$
0	16	= $\frac{1}{7}$	8	= $\frac{1}{7}$	4	= $\frac{1}{7}$	4	0	= $\frac{1}{5}$
0	14	= $\frac{1}{8}$	7	= $\frac{1}{8}$	2	= $\frac{1}{14}$	2	2	= $\frac{1}{8}$
0	8	= $\frac{1}{14}$	4	= $\frac{1}{14}$			2	0	= $\frac{1}{10}$
0	7	= $\frac{1}{16}$					1	1	= $\frac{1}{16}$
0	4	= $\frac{1}{28}$					1	0	= $\frac{1}{20}$

A Table of Discount per cent.

£	s.d.		£	s.d.		£	s.d.	
$1\frac{1}{4}$ per cent.	=0 3	On the Pound.	$8\frac{3}{4}$ per cent.	=1 9	On the Pound.	$22\frac{1}{2}$ per cent.	=4 6	On the Pound.
$2\frac{1}{2}$ ———	=0 6		10 ———	=2 0		25 ———	=5 0	
$3\frac{3}{4}$ ———	=0 9		$12\frac{1}{2}$ ———	=2 6		30 ———	=6 0	
5 ———	=1 0		15 ———	=3 0		35 ———	=7 0	
$6\frac{1}{4}$ ———	=1 3		$17\frac{1}{2}$ ———	=3 6		40 ———	=8 0	
$7\frac{1}{2}$ ———	=1 6		20 ———	=4 0		45 ———	=9 0	
						50 ———	=10 0	

☞ Practice admits of a great variety of rules, the multiplication of which serves little else than to confuse the mind of the scholar. The most useful are here given, and all that are actually necessary for the man of business; but the scholar who wishes to pursue his studies beyond this elementary treatise, will find the subject amply treated of in the large Arithmetick of Pike.

CASE I.

To find the value of any number of pounds, &c. when the price of one pound is an aliquot part of a dollar.

Rule.—As the value of any number of pounds at one dollar a pound, will be just so many dollars as there are pounds: therefore, call the pounds dollars, and divide them by such an aliquot part as the price is of a dollar, the quotient will be the answer in dollars.

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248. *How do you find the value of any number of pounds, &c. when the price per lb. is an aliquot part of a dollar.*

EXAMPLES.

1. What will be the value of 360 yards of cloth, at 25 cents a yard?

cts. $
25 = ¼)360

$90 Ans.

2. What will be the value of 75 pounds of loaf sugar, at 20 cents a pound?

cts. $
20 = ⅕)75

$15 Ans.

3. What will be the value of 125 pounds of butter, at 12½ cents a pound?

Ans. $15.625.

4. What will 1250 pounds of cheese come to, at 6¼ cents a pound?

Ans. $78.125.

CASE II.

To find the value of any number of yards, &c. when the price of one yard is an aliquot part of a pound.

Rule.—Call the number of yards so many pounds, and divide them by such an aliquot part as the price is of a pound, the quotient will be the answer in pounds.

EXAMPLES.

1. What will 63 gallons of rum come to, at 6s. 8d. a gallon?

s. d. £
6 8 = ⅓)63

£21 Ans.

2. What is the value of 125 yards of linen, at 5s. a yard?

s. £
5 = ¼)125

£31 5s. Ans.

3. What is the value of 500 bushels of potatoes, at 2s. 6d. a bushel?

Ans. £62 10s.

4. What will 37½ yards of sheeting come to, at 1s. 8d. a yard?

Ans. £3 2s. 6d.

5. What will be the value of 150 bushels of corn at 3s. 4d. a bushel?

Ans. £25.

6. What will 120 gallons of wine come to, at 10 shillings a gallon?

Ans. £60.

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249. *How do you find the value of a certain number of yards, when the price of one is an aliquot part of a pound?*

CASE III.

To find the value of any number of pounds, &c. when the price of one pound is an aliquot part of a shilling.

Rule.—Call the given number of pounds so many shillings, and divide them by such an aliquot part as the price is of one shilling, the quotient will be the answer in shillings.

EXAMPLES.

1. What will be the value of 650 pounds of cheese, at 6 pence per pound?

d. s.
$6 = \frac{1}{2})650$
$2|0)32|5$
£16 5s. Ans.

2. What will 1864 pounds of pork come to, at 4 pence a pound?

d. s.
$4 = \frac{1}{3})1864$
$2|0)62|1$—4d
£31 1s. 4d. Ans.

3. What is the value of 37½ yds. of ribband, at 3 pence a yard?
Ans. 9s. 4½d.

4. What will 75½ yards of tape come to, at 1½d. a yard?
Ans. 9s. 5¼d.

CASE IV.

To find the value of any number of yards, &c. when the price of one yard is shillings, or shillings, pence, and farthings, and not an aliquot part of a pound.

Rule.—Write down the given number of yards, &c. and call them so many pounds. Divide the given price into aliquot parts, either of a pound, or of one another; then divide by the several aliquot parts, and the sum of the quotients will be the answer in pounds.

EXAMPLES.

1. What will be the value of 36 yards of silk vesting, at 7s. 6d. a yard?

s. £
$5 = \frac{1}{4})36$
$2s.6d. = \frac{1}{2})9$
4 10s.
£13 10s. Ans.

2. What will 72 yards of Cassimere come to, at 15 shillings a yard?

s. £
$10 = \frac{1}{2})72$
$5 = \frac{1}{2})36$
18
£54 Ans.

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250. *How, when the price of a pound is an aliquot part of a shilling*

3. What is the value of 139 gallons of wine, at 9s. 10d. a gallon?
Ans. £68 6s. 10d.

4. What will 49½ pounds of tea come to, at 3s. 11½d. a pound?
Ans. £9 15s. 11¼d.

5. What will 146 yards of broadcloth come to, at 14s. 9d. a yard?
Ans. £107 13s. 6d.

6. What will 120 barrels of apples come to, at 11s. 3d. a barrel?
Ans. £67 10s.

CASE V.

To find the value of any number of pounds, &c. when the price of one pound is pence, or pence and farthings, and not an aliquot part of a shilling.

Rule.—Write down the given number of pounds, &c. and call them shillings. Divide the given price into aliquot parts, either of a shilling, or of one another; then divide by the several aliquot parts, and the sum of the quotients will be the answer in shillings.

EXAMPLES.

1. What will be the value of 56 pounds of raisins, at 8 pence a pound?

```
 d.     s.
 6 = ½)56
        —
 2 = ⅓)28
        9 4
        —
  2|0)3|7 4
        —
     £1 17s. 4d. Ans.
```

2. What will 48½ yards of riband come to, at 7½ pence a yard?

```
 d.      s. d.
 6  = ½)48 6
        ————
 1½ = ¼)24 3
         6 0¾
        —————
   2|0)3|0 3¾
        —
     £1 10s. 3¾d. Ans.
```

3. What is the value of 541 yards of cotton shirting, at 9¼d. a yard?
Ans. £20 17s. 0¼d.

4. What will 672 pounds of butter come to, at 11¾ pence a pound?
Ans. £32 18s.

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251. *How do you proceed, when the price is not an aliquot part of a pound, or of a shilling?*

CASE VI.

To find the value of any quantity consisting of several denominations, when the price of one hundred weight, &c. is given in several denominations.

Rule.—Multiply the price of a cwt. &c. by the number of hundreds. Divide the remainder of the quantity into aliquot parts either of a cwt or of one another; then divide by the several aliquot parts, and the sum of the product and quotients will be the answer.

EXAMPLES.

1. What will be the value of 5 cwt. 3 qrs. 14 lbs. of sugar at $9·56 a cwt.?

qrs.	$ cts.
2 = $\frac{1}{2}$	9·56
	5
	47·80
1qr.=$\frac{1}{2}$	4·78
14lbs.=$\frac{1}{2}$	2·39
	1·195
	$56·165 Ans.

2. What will 12 cwt. 1 qr. 16lbs. of hay come to, at 4 shillings 6 pence a cwt.?

	£	s.	d.
1 qr. = $\frac{1}{4}$		4	6
			12
	2	14	0
14 lbs. = $\frac{1}{2}$		1	1$\frac{1}{2}$
2 lbs. = $\frac{1}{7}$		0	6$\frac{3}{4}$
			0$\frac{3}{4}$ $\frac{6}{7}$
	£2	15	9 0$\frac{6}{7}$ Ans.

3. What cost 12 cwt. 0 qrs. 7 lbs. at $6·34 a cwt.?
Ans. $76·476$\frac{1}{4}$.

4. What is the value of 14 cwt. 3 qrs. 7 lbs. at 13s. 8d. a cwt.?
Ans. £10 2s. 5$\frac{1}{4}$d.

5. What is the value of a piece of broadcloth measuring 36 yards 3 qrs. at $7·75 a yard?
Ans. $284·8125.

6. What is the value of a farm containing 144 acres, 3 roods, 25 poles, at £6 10s. an acre?
Ans. £941 17s. 9$\frac{3}{4}$d.

---

252. *How will you get the value of a quantity of several denominations, when the price of one cwt. &c. is given in different denominations?*

FELLOWSHIP.

Fellowship is a rule by which the accounts of several merchants or other persons, trading in partnership, are so adjusted, that each may have his share of the gain, or sustain his share of the loss, in proportion to his share of the joint stock and the time of its continuance in trade.

Note. By this rule, a bankrupt's estate may be divided among his creditors.

SINGLE FELLOWSHIP.

Single Fellowship is, when the stock of each of the several partners is continued in trade the same length of time.

RULE.

As the whole amount of stock, is to the whole gain or loss, so is each man's stock, to his share of the gain or loss.*

Proof.—Add all the shares of the gain or loss together ; and if the work be right, the sum will be equal to the whole gain or loss.

EXAMPLES.

1. A, B, and C traded in partnership. A put into the stock 400 dollars, B put in 300 dollars, and C put in 200 dollars ; they gained 270 dollars : what was each man's share of the gain in proportion to what he put in ?

---

* That their gain or loss, in this Rule, is in proportion to their stocks, is evident : For as the times, in which the stocks are in trade, are equal, if I put in one half of the whole stock, I ought to have one half of the gain ; if my part of the stock be one fourth, my share of the gain or loss ought to be one fourth also. And generally the same ratio that the whole stock has to the whole gain or loss, must each person's particular stock have to his respective gain or loss.

---

253. *What is Fellowship ?*——254. *What is Single Fellowship ?*——255. *What is the rule ; and upon what principle is it founded ?*——256. *What is the method of proof ?*

$
400 A‘s stock.
300 B‘s stock.
200 C‘s stock.

$$\text{As } \$900 : \overset{\$}{270} :: \overset{\$}{400}$$

400

9|00)1080|00

$120 A‘s gain.

$$\text{As } \overset{\$}{900} : \overset{\$}{270} :: \overset{\$}{300}$$

300

9|00)810|00

$90 B‘s gain.

$$\text{As } \overset{\$}{900} : \overset{\$}{270} :: \overset{\$}{200}$$

200

9|00)540|00

$ 60 C‘s gain.

$$\text{Ans.} \left\{ \begin{array}{r l} \$120 & \text{A‘s gain.} \\ 90 & \text{B‘s gain.} \\ 60 & \text{C‘s gain.} \\ \hline 270 & \text{Proof.} \end{array} \right.$$

2. Divide the number 360 into four equal parts, which shall be to each other, as 3, 4, 5, and 6.

$$\text{As } 3+4+5+6 : 360 :: \left\{ \begin{array}{r r} 3: & 60 \\ 4: & 80 \\ 5: & 100 \\ 6: & 120 \end{array} \right\} \text{Answer.}$$

360 Proof.

3. A gentleman dying, left two sons and a daughter, to whom he bequeathed the following sums, viz. To the first son he gave 1200 dollars, to the second 1000 dollars, and to the daughter, 800 dollars; but it was found, that his whole estate amounted only to 750 dollars; what must each child receive of the estate, in proportion to the legacies?

$$\text{Answer,} \left\{ \begin{array}{l} 300 \text{ first son‘s portion.} \\ 250 \text{ second son‘s portion.} \\ 200 \text{ daughter‘s portion.} \end{array} \right.$$

4. A, B, and C traded in partnership. A put in 385 dollars, 50 cents; B put in 297 dollars, 75 cents; and C put in 175 dollars, 25 cents; they gained 343 dollars, 40 cents; what was each one‘s share of the gain?

$$\text{Answer,} \left\{ \begin{array}{r r l} \$154 & 20 & \text{A‘s gain.} \\ 119 & 10 & \text{B‘s gain.} \\ 70 & 10 & \text{C‘s gain.} \end{array} \right.$$

DOUBLE FELLOWSHIP.

Double Fellowship, or Fellowship with time, is when the stocks of partners are continued unequal times.

Rule.—Multiply each man's stock by the time it was continued in trade. Then, As the whole sum of the products is to the whole gain or loss, so is each man's particular product to his particular share of the loss or gain.*

EXAMPLES.

1. A, B, and C traded in company. A put in 400 dollars, for 9 months; B put in 300 dollars, for 6 months; C put in 200 dollars for 5 months: they gained 320 dollars; what was each man's share of the gain?

```
  $             $      $          $      $       $
 400           300    200    As 6400 : 320 : : 1800
 9             6      5                1800
----          ----   ----              ----
$3600 A's pr. 1800   1000              256
 1800 B's pr.                          32
 1000 C's pr.                         -----
 ----      $      $             64|00)5760|00($90 B's gain.
As 6400 : 320 : : 3600                 576
          3600                         ----
          ----                           0
          192                     $      $       $
          96                 As 6400 : 320 : : 1000
         -----                         1000
64|00)11520|00($180 A's gain.         -----
          64                 64|00)3200|00($50 C's gain.
          --                         320
          512                        ---
          512                         0
          ---
           0
```

```
      ($180 A's gain.
Ans. <  90 B's gain.
      (  50 C's gain.
       ----
       $320 Proof.
```

---

* When the stock of each partner is employed in trade the same length of time, the share of the gain or loss is, evidently, in proportion to the stock; and when the stock of each partner is the same, but employed in trade a different length of time, the share of the gain or loss is in proportion to the time; but when neither are the same, the share of the gain or loss of each partner must, evidently, be in proportion to the product of the stock multiplied by the time.

257. *What is Double Fellowship?*——258. *What is the rule?*——259. *Explain the difference between the two kinds of Fellowship?*

2. Two merchants entered into partnership for 18 months; K first put into stock \$400, and at the end of 8 months he put in \$200 more; L, at first, put in \$1100, and at the end of 4 months took out \$280.—Now at the expiration of the time, they found they had gained \$1052: What is each man's just share?

Ans. K, \$385·90. L, \$666·10.

3. A, B, and C, hold a pasture in common, for which they pay £20 per annum. In this pasture, A had 40 oxen for 76 days; B had 36 oxen for 50 days, and C had 50 oxen for 90 days. I demand what part each of these tenants ought to pay of the £20.

	£	s.	d.	q.	
Ans.	6	10	2	$1\frac{3340}{9340}$	A's part.
	3	17	1	$0\frac{2000}{9340}$	B's part.
	9	12	8	$2\frac{5000}{9340}$	C's part.

CUSTOM HOUSE ALLOWANCES.

The only allowances which are made in the weight of goods, at the Custom-Houses of the United States, are *tare*, and *draft* or *scalage*.

Tare is an allowance made for the weight of the box, bag, hogshead, cask, &c. which contains the goods, and it is either at so much per cent. or at so much per box, &c. or the real or actual tare.

Draft or *scalage* is an allowance of ½ per cent. on the whole gross weight of all goods, except tea and sugar, which is to be deducted before casting the tare.

On tea, no draft is allowed. On sugar, there is a deduction to be made for draft or scalage, before casting the tare, of 7 pounds on every hogshead, 4 pounds on every tierce, 2 pounds on every barrel, and 4 pounds on every Havanna box.

Gross weight is the whole weight of any kind of goods, together with the box, bag, or cask, &c. which contains them.

Net weight is the weight of the goods after all allowances have been deducted.

---

260. *What allowances are made at the American custom houses?*—261. *What is tare? draft or scalage? gross weight? and net weight?*

CASE I.

To find the net weight of goods, when the tare is at so much per cent., with the allowance for draft or scalage.

RULE.

1. When the scalage is ½ per cent. divide the whole gross weight by 200, and the quotient will be the scalage.

2. When the scalage is so much per hogshead, box, &c. multiply the scalage of one by the number of hogsheads, boxes, &c. and the product will be the whole scalage.

3. Subtract the scalage from the gross weight, then multiply the remainder by the tare per cent. and divide the product by 100, the quotient will be the whole tare, which subtract from what remained after the scalage was deducted, and the remainder will be the net weight.*

EXAMPLES.

1. A merchant bought 15 boxes of brown Havanna sugar, weighing 6740 lbs. gross, tare 15 per cent., scalage 4 lbs. per box; what was the net weight of the sugar, and what was its value at $10·25 per hundred weight?

Ans. $\begin{cases} \text{5678 lbs} = \text{50 cwt. 2 qrs. 22 lbs. net weight.} \\ \$519{\cdot}638\frac{11}{28} \text{ value.} \end{cases}$

2. What is the net weight of 4 bags of coffee, weighing 480 lbs. gross, allowing 2 per cent. for tare, and ½ per cent. for scalage; and what is the value of the net weight, at ·18 cents per pound?

Ans. $\begin{cases} \text{468 lbs. net weight.} \\ \$84{\cdot}24 \text{ value.} \end{cases}$

CASE II.

To find the net weight of goods, when the tare is at so much per box, cask, chest, &c. either with or without the allowance for draft or scalage.

---

* In casting tare and scalage, remainders, which do not exceed half a pound, are not usually reckoned, but if they exceed half a pound, then another pound is to be reckoned.

---

262. *What is the rule to find the net weight of goods, when the tare is at so much per cent. with an allowance for draft or scalage?*

RULE.

1. If draft or scalage be allowed, cast and deduct it as directed in Case 1st.

2. Multiply the tare of one by the number of boxes, chests, &c. the product will be the whole tare, which subtract from what remained after the scalage was deducted, or from the whole gross weight, when no scalage is allowed, and the remainder will be the net weight.

EXAMPLE.

1. What is the net weight and value of 4 casks of raisins weighing 520 lbs. gross, tare 12 lbs. per cask, allowing ½ per cent. for draft or scalage, at 7½ cents per pound ?

Ans. { 469 lbs. net weight.
$35·175 value.

CASE III.

To find the net weight of goods, when the real or actual tare is allowed.

RULE.

Subtract the whole amount of tare from the whole gross weight, and the remainder will be the net weight.

EXAMPLES.

1. What is the net weight and value of 50 boxes of figs, weighing 1200 lbs. gross, whole amount of tare 175 lbs. with an allowance of ½ per cent. for scalage, at ·10 cents per pound ?

Ans. { 1069 lbs. net weight.
$106·90 value.

2. What is the net weight and value of 4 firkins of butter, weighing as follows :

No. 1, gross 75 lbs. tare 12 lbs.
2, do. 83 lbs. do. 14 lbs.
3, do. 98 lbs. do. 17 lbs.
4, do. 120 lbs. do. 21 lbs.
at ·15 cents per pound ?

Ans. { 312 lbs. net weight.
$46·80 value.

---

263. *What is the rule, when the tare is at so much* per box, *&c. either with or without allowance ?*——264. *How do you find the weight of goods, when the real tare is allowed ?*

AMERICAN DUTIES.

DUTIES are imposed by law on goods, wares, and merchandise, imported into the United States, at certain rates per centum ad valorem, or at certain rates per ton, hundred, pound, gallon, &c.

The ad valorem rates of duty upon goods, wares and merchandise, are estimated by adding 20 per cent. to the actual cost thereof, if imported from the Cape of Good Hope, or from any place beyond it ; and 10 per cent. if imported from any other place or country, including ~~all charges ;~~ commissions, outside packages, and insurance excepted.

CASE I.

To find the duty on any amount of goods, wares or merchandise, at any given rate per centum ad valorem.

RULE.—Reduce the cost of the goods to Federal money, add 20 per cent. if imported from or beyond the Cape of Good Hope, or 10 per cent. if imported from any other place, then multiply the amount by the given rate per cent. and divide the product by 100, the quotient will be the duty required.

EXAMPLES.

1. What will be the duty on an invoice of woollen goods, imported from London, which cost £1250 10s. sterling, at 30 per cent. ad valorem ?

£1250.5 sterling, cost.
$4·44=£1 sterling.*

50020
50020
50020

$5552.220 actual cost in Federal Money.
555.222 ten per cent. added.

$6107.442 amount.
30

Ans. $1832.23260 duty required.

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* The rates at which foreign coins and currencies are estimated at the Custom Houses of the United States may be found on page 39.

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265. *How are duties imposed upon imported goods ?*——266. *In what manner are the ad valorem rates estimated ?*——267. *What is the rule for finding the duty on any amount, at a given rate* per cent. *ad valorem ?*

2. What will be the duty on an invoice of silk goods, imported from France, which cost 2650 Francs, at 20 per cent. ad valorem?
Ans. $109.31¼

3. What will be the duty on an invoice of silk and cotton goods, imported from India, which cost 25000 rupees at 25 per cent. ad valorem? Ans. $3750.

CASE II.

To find the duty on any amount of goods, wares, or merchandize, at any given rate per pound, gallon, &c.

Rule.—Multiply the given rate per pound, gallon, &c. by the number of pounds, gallons, &c. and the product will be the duty required.

EXAMPLES.

1. What will be the duty on 150 chests of Hyson tea imported direct from China in a vessel of the United States, weighing gross 11250 lbs. tare 20 lbs. per chest, at 40 cents per pound?
Ans. $3300.

2. What will be the duty on 20 pipes of French brandy, 4th proof, containing 2520 gallons, at 48 cents per gallon?
Ans. $1209.60.

3. What will be the duty on 25 hogsheads of brown sugar, weighing 43750 lbs. gross, allowing 12 per cent. tare, and 7 pounds per hogshead for draft or scalage, at 03 cents per pound?
Ans. $1150.38.

DISCOUNT.

Discount is an allowance made for the payment of any sum of money before it becomes due; or it is the difference between any sum due at some future time, and its present worth.

The *present worth* of any sum, or debt, due at any future time, is such a sum, as if put at interest, would in that time and at the rate per cent for which the discount is to be made, amount to the given sum or debt.

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268. *How do you find the duty, at any given rate per pound, &c.?*——269. *What is Discount?*——270. *What is the present worth of a sum due at a future time?*

RULE.

1. As the amount of \$100 or £100 at a given rate and time, is to the interest of \$100 or £100 at the same rate and time, so is the given sum to the discount.

Subtract the discount from the given sum, and the remainder is the present worth.

2. As the amount of \$100 or £100 at the given rate and time, is to \$100 or £100, so is the given sum to the present worth.

Subtract the present worth from the given sum, and the remainder is the discount.*

EXAMPLES.

1. What is the discount and present worth of \$500 payable in 8 months, allowing discount at 6 per cent. per annum?

By Rule 1.

$
100
Int. for 8 mo. 4
Amount 104

\$ \$ \$
104 : 4 : : 500
500

\$ c.
104)2000(19.23$\frac{1}{13}$ discount.
104
480.76$\frac{12}{13}$ pres. worth.
960
936
2400
208
320
312
$\frac{8}{104}=\frac{1}{13}$

By rule 2.

$
100
Int. for 8 mo. 4
Amount 104*

\$ \$ 4
104 : 100 : : 500
500

\$ c.
104)50000(480.76$\frac{12}{13}$ present worth.
416
19.23$\frac{1}{13}$ discount.
840
832
8000
728
720
624
$\frac{96}{104}=\frac{12}{13}$

---

* That an allowance ought to be made for the payment of money before it becomes due, which is supposed to bear no interest until after it is due, is very reasonable; and this allowance ought to be such a sum, as, being put at interest until the debt becomes due, would amount to the interest of the debt for the same time.

The reason of the rule is evident from the nature of simple interest; for since the debt may be considered as the amount of some principal (called here the present worth) at a certain rate per cent. for the given time, that amount must be in the same proportion either to its principal or interest, as the amount of any other sum at the same rate and for the same time, is to its principal or interest.

---

271. *What are the rules of Discount?*——272. *What are the reasons upon which they are founded?*

2. What is the present worth of £150 payable in 3 months, allowing discount at 5 per cent. per annum?

Ans. £ 148 2s. 11d. 2$\frac{2}{9}$qrs.

Note.—The only correct principle of calculating discount is by the preceding rule; yet banks and commercial persons, in discounting notes, &c. deduct the interest for the time (including the days of grace) for the discount. When discount is made for present payment without any regard to time, the interest of the sum for one year is the discount.

BARTER.

Barter is the exchanging of one commodity for another, and teaches merchants so to proportion their quantities, that neither shall sustain loss.*

Proof.—By changing the order of the question.

RULE.

1. *When the quantity of one commodity is given with its value or the value of its integer, as also the value of the integer of some other commodity to be exchanged for it, to find the quantity of this commodity.* Find the value of the commodity of which the quantity is given, then find how much of the other commodity at the rate proposed may be had for that sum.

2. *When the quantities of two commodities are given, and the rate of selling them, to find, in case of inequality, how much of some other commodity must be given.*—Find the separate values of the two given commodities; subtract the less from the greater, and the difference will be the amount of the third commodity, whose quality and rate may be easily found.

---

* The rules of Barter are merely applications of the Rule of Three, and are easily understood.

273. What is Barter?——274. Explain to me the given rules, if you understand them.

3. *When, in bartering, one commodity is reckoned above the ready money price,—to find the bartering price of the other* :—Say, as the ready money price of the one, is to its bartering price ; so is *that* of the other, to *its* bartering price : Next, find the quantity required, according to either the bartering or ready money price.

EXAMPLES.

1. How much tea at 9s. 6d. per lb. must be given in barter for 156 gallons of wine, at 12s. $3\frac{1}{2}$d. per gallon ?

$$\begin{array}{r|r|r}
 & & \text{Galls.} \\
3\text{d.} & \frac{1}{4} & 156 \\
\frac{1}{2} & \frac{1}{8} & 12 \\
\hline
 & & 1872 \\
 & & 39 \\
 & & 6\ \ 6 \\
\hline
 & 9\text{s. }6\text{d.} = 114\text{d.} & 1917\ \ 6 \\
 & & 12 \\
\hline
 & & 23010
\end{array}$$

d. lb. d. lb. oz.
As 114 : 1 : 23010 : : 201 $13\frac{54}{114}$ Ans.

price. quan. price. quan.
s. d. gals. s. d. lb. oz.
Or, As 12 $3\frac{1}{2}$: 156 : 9 6 : 201 $13\frac{54}{114}$ Ans. as before.

2. A and B would barter; A has 150 bushels of wheat, at $1·25c. per bushel, for which B gives 65 bushels of barley, worth ·$62\frac{1}{2}$c. per bushel, and the balance in oats at ·$37\frac{1}{2}$c. per bushel. What quantity of oats must A receive from B ?

Ans. $391\frac{2}{3}$ bushels.

3. A has linen cloth, at ·30c. per yard, ready money, in barter ·36c. ; B has 3610 yards of ribband, at .22c. per yard ready money, and would have of A $200 in ready money, and the rest in linen cloth : what rate does the ribband bear in barter per yard, and how much linen must A give B ?

Ans. The rate of ribband is ·26c. 4m. per yard, and B must receive $1980\frac{2}{3}$ yards of linen, and $200 in cash.

LOSS AND GAIN.

Loss and Gain is a rule by which merchants and traders find what they gain or lose by trading; and at what rate per cent.:—It also teaches them to find the price for which any kind of goods must be sold, in order to gain or lose any given rate per cent.—The different cases are only particular applications of the Rule of Three.

CASE I.

When goods are bought at one price, and sold at another, to find what is gained or lost, and the gain or loss per cent.

Rule.—Find the gain or loss by subtraction; then, As the price the goods cost, is to the gain or loss, so is $100 or £100 to the gain or loss per cent.

EXAMPLES.

1. If I buy cloth for $2 a yard, and sell it for $2·50 a yard; what do I gain per yard, and what do I gain per cent. or by laying out 100 dollars?

$ c.
Sold for 2·50
Cost 2·00
Gained ·50 per yard.

$ c. $
As 2 : ·50 : : 100
100
2)5000
Gained $25·00 per cent.

Note. When goods are bought or sold on credit, the present worth of the value of the goods for the time, must be found, in order to find the true gain or loss.

2. If I buy cloth at $3·50 per yard for cash, and sell it at $4·12 per yard on a credit of six months, what do I gain per cent. allowing discount at 6 per cent. a year on the selling price?

Ans. $14·28¼ per cent.

3. Bought 12 cwt. 2 qrs. of sugar at $10·20 per cwt. on a credit of 4 months, and sold the same at $10·25 per cwt. for cash; what was the whole gain, and the gain per cent. allowing discount at 6 per cent. a year on the purchase price?

Ans. { $3·12½ whole gain.
2·50 gain per cent.

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275. What is Loss and Gain?——276. What is the rule for finding loss or gain in business?——277. How do you proceed, when goods are bought or sold on credit?

CASE II.

To find the price for which any kind of goods must be sold, in order to gain or lose any given rate per cent.

Rule.—As $100 or £100 is to the purchase price, so is $100 or £100 with the profit per cent. added, or loss per cent. subtracted, to the selling price.

EXAMPLES.

1. Bought linen at ·60 cents per yard, how must it be sold per yard, in order to gain 25 per cent ?

$ c. $ c.
As 100 : ·60 : : 125 : ·75 Ans.

2. Bought a piece of cloth at $2·75 per yard, and sold it at a loss of 15 per cent. : what was it sold for per yard ?

Ans. $2·3375.

3. Bought 50 gallons of brandy, at ·75 cents per gallon, but by accident, 10 gallons leaked out : At what rate must I sell the remainder per gallon, to gain upon the whole prime cost, at the rate of 10 per cent. ?

Ans. $1·03c. 1¼m.

EQUATION OF PAYMENTS.

Equation of Payments is the finding of a time to pay at once, several debts due at different times, so that neither party shall sustain loss.

Rule.—Multiply each payment by the time at which it is due ; then divide the sum of the products by the sum of the payments, and the quotient will be the equated time.*

---

*This rule is founded on a supposition, that the sum of the interests of the several debts which are payable before the equated time, from their terms to that time, is equal to the sum of the interests of the debts payable after the equated time, from that to their terms ; but this is not correct, for by keeping a debt unpaid after it is due, the interest of it is gained for that time ; but by paying a debt before it is due, the payer does not lose the interest for that time, but the discount only, which is less than the interest ; therefore, the rule is not accurately true ; however, in most questions which occur in business, the errour is so trifling, that it will generally be made use of as the most eligible method.

278. *How do you ascertain at what price you must sell an article in order to gain so much per cent. ?*——279. *What is Equation of Payments ?*——280. *What is the rule ; and on what is it founded ?*

EXAMPLES.

1. A owes B $380 to be paid as follows, viz. $100 in 6 months, $120 in 7 months, and $160 in 10 months: What is the equated time for the payment of the whole debt?

100× 6= 600
120× 7= 840
160×10=1600

100+120+160=380)3040(8 months, Ans.
3040

2. A owes B £104 15s. to be paid in 4½ months, £161 to be paid in 3½ months, and £152 5s. to be paid in 5 months: What is the equated time for the payment of the whole?

Ans. 4 months and 8 days.

3. There is owing to a merchant £698, to be paid £178 ready money, £200 at 3 months, and £320 in 8 months; I demand the indifferent time for the payment of the whole?

Ans. 4½ months.

4. The sum of $164 16c. 6m. is to be paid, ½ in 6 months, ⅓ in 8 months, and ⅙ in 12 months: what is the mean time for the payment of the whole?

Ans. 7⅔ months.

5. A merchant has $360 due him, to be paid at 6 months, but the debtor agrees to pay ½ at the present time, and ⅓ at 4 months; I demand the time he must have to pay the remainder, at simple interest, so that neither party may have the advantage of the other?

½=180 paid down.
⅓=120 paid at 4 months.
⅙= 60 unpaid.

Now as he pays 180 dollars 6 months, and 120 dollars 2 months before they are respectively due, say, as the interest of 60 dollars for 1 month, is to 1 month, so is the sum of the interest of 180 dollars for 6 months, and of 120 dollars for 2 months, to a fourth number, which added to the 6 months, will give the time for which the 60 dollars ought to be retained.

Ans. 28 months.

INVOLUTION.

INVOLUTION is the method of finding the powers of numbers.

Powers of numbers are the products arising from the continual multiplication of numbers into themselves.

Any number may itself be called the root or *first power*. If the first power be multiplied by itself, the product is called the *second power*, or *the square* ; if the square be multiplied by the first power, the product is called the *third power*, or the *cube* ; if the cube be multiplied by the first power, the product is called the *fourth power*, or the *biquadrate*, &c.

Thus 4 is the root or 1st power of 4.
$4\times4=16$ is the 2d power, or the square of 4. $=4^2$
$4\times4\times4=64$ is the 3d power, or the cube of 4. $=4^3$
$4\times4\times4\times4=256$ is the 4th power, or the biquadrate of 4, &c. $=4^4$

The small figure points out the order of the power, and is called the *Index*, or *Exponent*.

Rule for finding the powers of numbers.

Multiply the given number, or first power, continually by itself, till the number of multiplications be one less than the index of the power to be found, and the last product will be the power required.

NOTE.—The powers of vulgar fractions are found by raising each of their terms to the power required. If the power of a mixed number be required, either reduce it to an improper fraction, or reduce the vulgar fraction to a decimal.

* * *

EVOLUTION.

EVOLUTION, or the extraction of roots, is the operation by which we find any root of any given number.

The root is a number whose continual multiplication into itself produces the power, and is denominated the square, cube, biquadrate, or 2d, 3d, 4th root, &c. accordingly as it is, when raised to the 2d, 3d, 4th, &c. power, equal to that power. Thus, 4 is the square root of 16, because $4\times4=16$. 4 also is the cube root of 64, because $4\times4\times4=64$; and 3 is the square root of 9, and 12 is the square root of 144, and the cube root of 1728, because $12\times12\times12=1728$, and so on.

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281. *What is Involution ?*——282. *What are powers of numbers ?*——283. *How do you find these powers ?*——284. *What is Evolution ?*——285. *What is a root ?*

A TABLE OF POWERS.

Roots,	or 1st Pow'r	1	2	3	4	5	6	7	8	9
Squares,	or 2d Pow'r	1	4	9	16	25	36	49	64	81
Cubes,	or 3d Pow'r	1	8	27	64	125	216	343	512	729
Biquadrates,	or 4th Pow'r	1	16	81	256	625	1296	2401	4096	6561
Sursolids,	or 5th Pow'r	1	32	243	1024	3125	7776	16807	32768	59049
Square Cubes,	or 6th Pow'r	1	64	729	4096	15625	46656	117649	262144	531441
Second Sursolids,	or 7th Pow'r	1	128	2187	16384	78125	279936	823543	2097152	4782969
Biquadrates Sq'd.	or 8th Pow'r	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
Cubes Cubed,	or 9th Pow'r	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489
Sursolids Squar'd,	or 10th Pow'r	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401
Third Sursolids,	or 11th Pow'r	1	2048	177147	4194304	48828125	362797056	1977326743	8589934592	31380959609
Square Cubes Sqd.	or 12th Pow'r	1	4096	531441	16777216	244140625	2176782336	13841287201	68719476736	282429536481
Fourth Sursolids,	or 13th Pow'r	1	8192	1594323	67108864	1220703125	13060694016	96889010407	549755813888	2541865828329
2d Sursolids Sqd.	or 14th Pow'r	1	16384	4782969	268435456	6103515625	78364164096	678223072849	4398046511104	22876792454961
Sursolids Cubed,	or 15th Pow'r	1	32768	14348907	1073741824	30517578125	470184984576	4747561509943	35184372088832	205891132094649

There is no number of which we cannot find any power exactly; but there are many numbers, of which the exact roots can never be obtained. Yet, by the help of decimals, we can obtain these roots to any necessary degree of exactness.

Those roots which cannot be exactly obtained, are called *surd roots*; and those which can be found exactly, are called *rational roots*.

Roots are sometimes denoted by writing this character $\sqrt{}$ before the power, with the index of the power over it; thus the cube root of 64 is expressed $\sqrt[3]{64}$, and the square root of 64 is expressed $\sqrt{64}$, the index 2 being omitted when the square root is required.

EXTRACTION OF THE SQUARE ROOT.

THE EXTRACTION OF THE SQUARE ROOT is the method of finding a number, which, being multiplied by itself, shall produce the given number.

RULE.

1. Distinguish the given number into periods of two figures each, by putting a point over the place of units, another over the place of hundreds, and so on, which points show the number of figures the root will consist of.

2. Find the greatest square number in the first, or left hand period; place the root of it at the right hand of the given number, (after the manner of a quotient in division,) for the first figure of the root, and the square number under the period, then subtract it therefrom, and to the remainder bring down the next period for a dividend.

3. Place double of the root, already found, on the left hand of the dividend for a divisor.

4. Seek how often the divisor is contained in the dividend, (except the right hand figure,) and place the answer in the root for the second figure of it, and likewise on the right hand of the divisor; multiply the divisor with the figure last annexed by the figure last placed in the root, and subtract the product from the dividend: to the remainder join the next period for a new dividend.

---

286. *Can you find the power and root of any number?*——287. *What is the distinction in the roots?*——288. *What is the extraction of the Square Root?*——289. *What is the rule?*

5. Double the figures already found in the root, for a new divisor, (or bring down your last divisor for a new one, doubling the right hand figure of it,) and from these, find the next figure of the root as last directed, and continue the operation in the same manner, till you have brought down all the periods.*

---

*The rule for the extraction of the square root may be illustrated by attending to the process by which any number is raised to the square. The several products of the multiplication are to be kept separate, as in the proof of the rule for multiplication of simple numbers. Let 37 be the number to be raised to the square.

$$37\times37=1369=37\times37$$

$$\begin{array}{l}37\\ \hline 49=7^2\\ \left.\begin{array}{l}21=3\times7\\21=3\times7\end{array}\right\}=2\times3\times7\\ 9=3^2\\ \hline \qquad(37\\ 2\times3)42=2\times3\times7\\ \quad 49=7^2\end{array}
\qquad
\begin{array}{l}37\\ \hline 49=7^2\\ \left.\begin{array}{l}210=30\times7\\210=30\times7\end{array}\right\}=2\times30\times7\\ 900=30^2\\ \hline (30+7=37\end{array}$$

Now it is evident that 9, in the place of hundredths, is the greatest square in this product; put its root, 3, in the quotient, and 900 is taken from the product. The next products are $21+21=2\times3\times7$, for a dividend. Double the root already found, and it is 2×3. for a divisor, which gives 7 for the quotient, which annexed to the divisor, and the whole then multiplied by it, gives $2\times3\times7(=42)+7\times7(=49)$ which, placed in their proper places, completely exhausts the remainder of the square. The same may be shown in any other case, and the rule becomes obvious.

Perhaps the following method may be considered more simple and plain. Let $37=30+7$, be multiplied as in the demonstration of multiplication of simple numbers, and the products kept separate.

$$\begin{array}{l}30+7\\ 30+7\\ \hline 900+30\times7\\ \qquad\quad 30\times7+49\\ \hline 900+2\times30\times7+49=1316,\text{ the sum and square.}\\ 900 \qquad\qquad\qquad\qquad 30+7\\ \hline 2\times30+7\times7)2\times30\times7+49\\ \qquad\qquad\qquad\; 2\times30\times7+49\\ \hline \qquad\qquad\qquad\; *\;\;*\;\;\;*\;\;*\end{array}$$

The root of 900 is 30, and leaves the two other terms, which are exhausted by a divisor formed and multiplied as directed in the rule.

---

290. *Explain to me the nature of this rule.*

NOTE 1. If when the given power is pointed off, as the power requires, the left hand period should be deficient, it must nevertheless stand as the first period.

2. If there be decimals in the given number, it must be pointed both ways from the place of units: If when there are integers, the first period in the decimals be deficient, it may be completed by annexing so many ciphers as the power requires: And the root must be made to consist of so many whole numbers and decimals as there are periods belonging to each; and when the periods belonging to the given number are exhausted, the operation may be continued at pleasure by annexing ciphers.

EXAMPLES.

1. Required the square root of 729 ?

729(27 *the root.*
4
—
47)329
329
—
000

PROOF.

27
27
—
189
54
—
729

The given number being distinguished into periods, we seek the greatest square number in the left hand period (7) which is 4, of which the root (2) being placed to the right hand of the given number, after the manner of a quotient, and the square number (4) subtracted from the period (7) to the remainder (3) we bring down the next period (29) making for a dividend, 329. Then the double of the root (4) being placed to the left hand for a divisor, we say how often 4 in 32? (*excepting* 9 *the right hand figure*) the answer is 7, which we place in the root for the second figure of it, and also to the right hand of the divisor; then multiplying the divisor thus increased by the figure (7) last obtained in the root, we place the product underneath the dividend, and subtract it therefrom, and the work is done.

DEMONSTRATION OF THE REASON AND NATURE OF THE RULE.

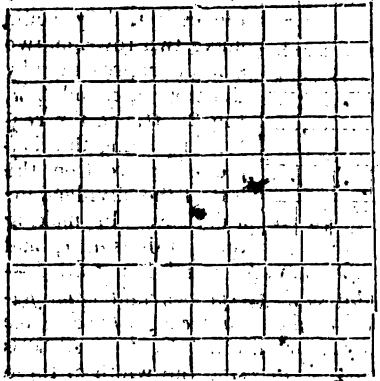
The superficial content of any thing, that is to say, the number of square feet, inches, &c. in the surface of a field, a floor, &c. is found by multiplying the length into the breadth. Thus, if a piece of land be 10 rods in length, and 10 in width, it is a square, and the measure of one of its sides is the root, of which the superficial content of the piece of land is the 2d power. Or, supposing you have a piece of cloth 1 yard wide, and 225 yards in length, and you wish to know how many square yards it will cover, you must so arrange the parts of the whole that they may be in a square form.

Now, suppose you have 144 square pieces of wood, and wish to know how many pieces would be on a side, were the whole arranged into a square form. To determine this, you must extract the square root 144; the first step of which is to point off the number into periods of two figures each. This shows how many figures the root will consist of, and is done on this principle, that the product of any two numbers can have, at most, but as many places of figures, as are in both the factors, and at least but one less.

. .
144(1
1
—
44

The left hand period being 1, the square of it will be 1, and likewise the root will be 1. But as we have nothing to do at present with the right hand period, we will omit it, and consider only the left hand period, which being in the place of hundreds, must be called 100; hence the operation, at present, will be to find the square root of 100. The root of 1, is 1, but as there are two periods in 100, there will be two figures in its root, and as the figure already obtained in the root is equal to its period, there is nothing remaining for the next period; and as the next period consists wholly of ciphers, the next figure of the root will be a cipher, so that the root of 100 is 10. By this process we have disposed of 100 of the pieces into the form represented by Fig. 1, viz. 10 pieces on a side.

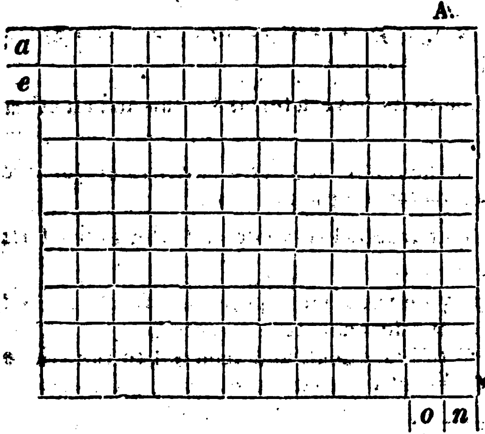
Fig. 1.



The reason for placing the square number underneath the period, and subtracting it from the period, as directed in the rule, is as follows. When we have obtained the root of the left hand period, we have disposed of as many pieces as the greatest square of the left hand period represents, and by subtracting the square of the root from its period, we make it smaller by as many as the square of the root represents; thus in the example given, 1 in the quotient represents 10, the square of which is 100, which 1, under the left hand period, represents. This, subtracted from the left hand period, leaves 44; so that 100 pieces have been disposed of as represented by Fig. 1, and 44 pieces are now to be added to it, in such manner that the square form will be preserved. To do this, the rule directs to "*place the double of the root already found on the left hand of the dividend for a divisor.*"

144(12
1
22)44
44

Fig. 2.



Now the first figure of the root shows the number of pieces there are on a side of Fig. 1, viz. 10. In order to preserve the square form, the additions must be made on two adjoining sides of the square, as in Fig. 2. Now it is evident that if there were just 20 pieces left, after disposing of 100, there would be just enough to make a row on two sides of Fig. 1, and if there were 40 pieces left, they would make two rows, on two sides, as represented by the rows *a*, *e*, and *o*, *n*, Fig. 2. Hence the reason of placing the double of the root on the left of the dividend for a divisor. In making the additions *a*, *e*, and *o*, *n*, you will observe there is a deficiency, A. which is not filled.—To fill this deficiency, the rule directs to "*except the right hand figure*," and likewise to "*place the quotient figure on the right hand of the divisor*." Now the deficiency A. must be limited by the additions *a*, *e*, and *o*, *n*, consequently the figure, expressing the width of these additions, expresses the root of this deficiency, which, multiplied into itself, gives the superficial contents of the deficiency. Thus Fig. 2 shows the disposition of 144 pieces into a square form.

2. Required the square root of 575·5 ?

575·50(23·98+, root.
4
43)175
129
469)4650
4221
4788)42900
38304
4596 remainder.

3. What is the square root of 10342656 ?

Ans. 3216.

4. What is the square root of 964·5192360241 ?

Ans. 31·05671.

5. What is the square root of ·0000316969 ?

Ans. ·00563.

Rules for extracting the Square Root of Vulgar Fractions and Mixed Numbers.

1. Reduce the fraction to its lowest terms for this and all other roots.

2. Extract the root of the numerator for a new numerator, and the root of the denominator for a new denominator.

3. Or reduce the vulgar fraction to a decimal, and extract its root.

4. Mixed numbers may be reduced to improper fractions, and the root of the numerator and denominator extracted, or the vulgar fraction may be reduced to a decimal, and annexed to the whole number, and the root of the whole extracted.

EXAMPLES.

1. What is the square root of $\frac{144}{15129}$?

$9)\frac{144}{15129}=\frac{16}{1681}$

16(4 root of the numerator.
16

1681(41 root of the denominator.
16

81)81 Therefore $\frac{4}{41}$ is the root of
81 [the given fraction.

Or, 1681)16·(·0095181439+. And $\sqrt{\cdot0095181439}=\cdot09756+$.

2. What is the square root of $\frac{2793}{8208}$?

Ans. $\frac{7}{12}$.

3. What is the square root of $42\frac{1}{4}$?

Ans. $6\frac{1}{2}$.

---

291. *What is the rule for extracting the Square Root of Vulgar Fractions and mixed numbers ?*

APPLICATION AND USE OF THE SQUARE ROOT.

Problem I. *To find a mean proportional between two numbers.*

Rule.—Multiply one of the given numbers by the other, and extract the square root of the product, and the root will be the mean proportional required.

Note. When the first number is as many times greater than the second, as the second is times greater than the third, the second number is called a mean proportional between the other two.

EXAMPLE.

1. What is the mean proportional between 36 and 144?

$36 \times 144 = 5184$, and $\sqrt{5184} = 72$ Ans.

Problem II. *To find the side of a* square *equal in* area *to any given* superficies *whatever.*

Rule.—Find the *area*, and the square root is the side of the square sought.*

EXAMPLES.

1. If the area of a triangle be 160, what is the side of a square equal in area thereto?

$\sqrt{160} = 12.649+$ Ans.

Problem III. A certain general has an army of 5625 men: pray, how many must he place in rank and file, to form them into a square?

$\sqrt{5625} = 75$ Ans.†

Problem IV. If a pipe 6 inches bore, will be 4 hours in running off a certain quantity of water, in what time will 3 pipes, each 4 inches bore, be in discharging double the quantity?

$6 \times 6 = 36$. $4 \times 4 = 16$, and $16 \times 3 = 48$. Then, as 36 : 4h. : : 48 : 3h. inversely, and as 1w. : 3h. : : 2w. : 6h. Ans.

---

* A square is a figure of four equal sides, each pair meeting perpendicularly, or, a figure whose length and breadth are equal. As the area, or number of square feet, inches, &c. in a square, is equal to the product of two sides which are equal, the second power is called the square. Hence the rule of **Problem II.** is evident.

† If you would have the number of men be double, triple, or quadruple, &c. as many in rank as in file, extract the square root of ½. ⅓. ¼. &c. of the given number of men, and that will be the number of men in file, which double, triple, quadruple, &c. and the product will be the number in rank.

292. How do you find a mean proportional between two numbers?

Problem V. A line 36 yards long will exactly reach from the top of a fort to the opposite bank of a river, known to be 24 yards broad. The height of the wall is required?

$36\times36=1296$; and $24\times24=576$. Then, $1296-576=720$, and $\sqrt{720}=26\cdot83+$yards, the Answer.

Problem VI. The height of a tree growing in the centre of a circular island 44 feet in diameter, is 75 feet, and a line stretched from the top of it over to the hither edge of the water, is 256 feet. What is the breadth of the stream, provided the land on each side of the water be level?

$256\times256=65536$; and $75\times75=5625$: Then, $65536-5625=59911$ and $\sqrt{59911}=244\cdot76+$ and $244\cdot76-\frac{44}{2}=222\cdot76$ feet, Answer.

Problem VII. Suppose a ladder 60 feet long be so planted as to reach a window 37 feet from the ground, on one side of the street, and without moving it at the foot, will reach a window 23 feet high on the other side; I demand the breadth of the street?

Ans. 102·64 feet.

Problem VIII. *Given the difference of two numbers, and the difference of their squares, to find the numbers.*

Rule.—Divide the difference of the squares by the difference of the numbers, and the quotient will be their sum. Then proceed by Prob. 4, p. 57.

EXAMPLES.

1. The difference of two numbers is 20, and the difference of their squares is 2000; what are the numbers?

Ans. 60 the greater. 40 the less.

2. Said Harry to Charles, my father gave me 12 apples more than he gave brother Jack, and the difference of the squares of our separate parcels was 288: Now, tell me how many he gave us, and you shall have half of mine.

Ans. { Harry's share 18
Jack's share 6

EXTRACTION OF THE CUBE ROOT.

A **Cube** is any number muliplied by its square. To extract the cube root, is to find a number which being multiplied into its square, shall produce the given number.

RULE.

1. Separate the given number into periods of three figures each, by putting a point over the unit figure and every third figure beyond the place of units.

2. Find the greatest cube in the left hand period, and put its root in the quotient.

3. Subtract the cube thus found, from the said period, and to the remainder bring down the next period, and call this the dividend.

4. Multiply the square of the quotient by 300, calling it the triple square, and the quotient by 30, calling it the triple quotient, and the sum of these call the divisor.

5. Seek how often the divisor may be had in the dividend, and place the result in the quotient.

6. Multiply the triple square by the last quotient figure, and write the product under this dividend; multiply the square of the last quotient figure by the triple quotient, and place this product under the last; under all, set the cube of the last quotient figure and call their sum the subtrahend.

7. Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before, and so on till the whole be finished.*

---

* The process for extracting the cube root may be illustrated in the same manner as that for the square root. Take the same number 37, and multiply as before, collecting the twice 21 into one sum, as they belong to the same place, and the operation will be simplified, $37^3=50653$.

$$37^2=\left\{\begin{array}{l} 49=7^2 \\ 42.=2\times3\times7 \\ 9\cdot\cdot=3^2 \end{array}\right.$$

$$\begin{array}{l} 49=7^2 \\ 420=2\times30\times7 \\ 900=30^2 \\ \hline 37 \end{array}$$

37 the multiplier.

---

293. *What is a Cube?*——294. *What is the method of extracting the cube root of a given number?*——295. *I wish you to illustrate the process under this rule, by one of the examples given.*

Note.—The same rule must be observed for continuing the operation and pointing for decimals, as in the square root.

EXAMPLES.

1. Required the cube root of 436036824287.

$$\begin{array}{rl}
 & \dot{4}3\dot{6}03\dot{6}82\dot{4}28\dot{7}(7583 \text{ the root.} \\
 & 343 \\ \hline
\text{1st Divisor}=14910) & 93036=\text{1st Dividend.} \\ \hline
 & 73500 \\
 & \;\;5250 \\
 & \;\;\;\;125 \\ \hline
 & 78875=\text{1st Subtrahend.}
\end{array}$$

$$37^3=\left\{\begin{array}{l}
343=7^3 \\
294=2\times3\times7^2 \\
63=3^2\times7 \\
147=3\times7^2 \\
126=2\times3^2\times7 \\
27=3^3
\end{array}\right.
\qquad
\begin{array}{l}
343=7^3 \\
2940=2\times30\times7 \\
6300=30^2\times7 \\
1470=30\times7^2 \\
12600=2\times30^2\times7 \\
27000=30^3
\end{array}$$

$$\begin{array}{rl}
 & 27 \quad (37 \\ \hline
3\times3^2\;) & 189=3\times3^2\times7 \\
3\times3\;) & 441=3\times3\times7^2
\end{array}
\qquad
\begin{array}{rl}
 & 27000 \quad (30+7 \\ \hline
3\times30^2\;) & 18900=3\times30^2\times7 \\
3\times30\;) & 4410=3\times30\times7^2
\end{array}$$

As 27 or 27000 is the greatest cube, its root is 3 or 30, and that part of the cube is exhausted by this extraction. Collect those terms which belong to the same places, and we have $3^2\times7=63$, and $2\times3^2\times7=126$, and $63+126=3\times3^2\times7=189$; and $2\times3\times7^2=294$ and $3\times7^2=147$, and $294+147=441=3\times3\times7^2$ for a dividend, which divided by the divisor, formed according to the rule, the quotient is 7, for the next figure in the root.

And it is evident, on inspecting the work, that that part of the cube not exhausted is composed of the several products which form the subtrahend, according to the rule.—The same may be shown in any other case, and the universality of the rule hence inferred.

The other method of illustration, employed in the square root, is equally applicable in this case.

$$\begin{array}{rl}
37=30+7, \text{ and } 30+7^2= & 30^2+2\times30\times7+7^2 \\
 & 30\;+7 \text{ the multiplier.} \\ \hline
 & 30^3+2\times30^2\times7+30\times7^2 \\
 & \qquad\quad 30^2\times7+2\times30\times7^2+7^3 \\ \hline
37^3=50653= & 30^3+3\times30^2\times7+3\times30\times7^2+7^3\,(30+7=37 \\
 & 30^3 \\ \hline
\text{Divisor } 3\times30^2+3\times30 \quad &)3\times30^2\times7+3\times30\times7^2+7^3 \text{ div.} \\ \hline
 & 3\times30^2\times7+3\times30\times7^2+7^3 \text{ subtrahend.}
\end{array}$$

It is evident that 30^3 is the greatest cube. When its root is extracted, the next three terms constitute the dividend; and the several products formed by means of the quotient or second figure in the root, are precisely equal to the remaining parts of the power whose root was to be found.

2d Divisor=1689750)14161824=2d Dividend.
13500000
144000
512

13644512=2d Subtrahend.

3d Divisor=172391940)517312287=3d Dividend.
517107600
204660
27

517312287=3 Subtrahend.

The Method of Operation.

7×7×300 = 14700 = 1st Triple square.
7×30 = 210 = 1st Triple quotient.

14910 = 1st Divisor.

14700×5 = 73500
5×5×210 = 5250
5×5×5 = 125

78875 = 1st Subtrahend.

75×75×300 = 1687500 = 2d Triple square.
75×30 = 2250 = 2d Triple quotient.

1689750 = 2d Divisor.

1687500×8 = 13500000
2250×8×8 = 144000
8×8×8 = 512

13644512 = 2d Subtrahend.

758×758×300 =172369200 = 3d Triple square.
758×30 = 22740 = 3d Triple quotient.

172391940 = 3d Divisor.

172369200×3 =517107600
22740×3×3 = 204660
3×3×3 = 27

517312287 = 3d Subtrahend.

DEMONSTRATION OF THE REASON AND NATURE OF THE RULE.

A block of wood, or any solid body, having *six equal sides*, all exactly square, is a CUBE; the root of which is the measure in length of one of its sides. To gain a distinct understanding of the subject, let the scholar provide himself with little blocks of wood, and build them up into a cubick form, according to the rule.

First make a cubick block of any given size, and mark it with the letter A. Then make three other blocks of a *square* form, of an indefinite thickness, but all equal to each other, each of which will just cover one side of the block A, and mark them B, C and D. Place these blocks on three adjoining sides of the block A, when there will be deficiencies at the three points where the blocks B, C and D meet. These deficiencies must be filled with three other blocks, each of which must be just equal in length to one side of the block A, and mark these blocks with E, F and G. When the blocks E, F, G are put in their places, there will be a deficiency at the place where the ends of these blocks meet. This deficiency must be filled with another block, which mark H. To illustrate the rule, take the following number.

```
10648(2
8
—
2
```

In the first place I seek the greatest cube in the left hand period, and place its root, 2, in the quotient. The cube of 2 is 8, which I place under the left hand period, and subtract it therefrom, which leaves a remainder of 2. Now as there are two periods in the given number, there must be 2 figures in the root, consequently, 2, in the quotient, does not express 2, merely, but 20; and the cube of 20 is 8000, which 8, under the period 10, represents; thus 8000 of the parts of 10648, are disposed of into a cubick body, the length of each side of which is equal to 20 of those parts, and to render the explanation more plain, we will consider these parts as cubick feet, so that each side of this body is 20 feet square, and this body we will have represented by the block A. Now as each side of this block is 20 feet square, there are 400 feet on each side of it. Now 8000 feet are disposed of in this block, consequently, there are 2648 cubick feet to be added to the block A, in such a manner, that its cubick form will be preserved. To do this, the additions must be made to three sides of the block, and these additions are represented by the blocks B, C and D, each of which containing 400 feet, the sum of the whole is 1200. Thus it is evident, that if there were 1200 feet more, there would be just enough to cover three sides of the block A; and it is to find the contents of these three sides, that the rule directs to "*multiply the square of the quotient by* 300." The square of the quotient shows the superficial contents of one side of the block A. viz. 400, for 2 in the quotient is in reality 20, and 20×20=400, and it is because the cipher is not annexed to the quotient figure, that we are directed to multiply the square of the quotient by 300 instead of 3, as in the following work.

```
    4                          10648(2       1200      60
  300                           8               2       4
 ----                          -------       ----    ----
 1200 triple square.       1260)2648         2400     240
                                2400
        2                        240
       30                          8 cube of the root.
       --                       ----
       60 triple quotient.      2648 subtrahend.
     1200                       ----
     ----
     1260 divisor.
```

The rule directs to "*multiply the quotient by 30.*" This is to obtain the contents of the blocks E, F and G, and the sum of these is taken for a divisor, because the number of times the dividend contains the divisor, will be equal to the number of feet, the additions to the block, A. are in thickness.

The rule next directs to "*multiply the triple square by the last quotient figure.*" Now the triple square represents the superficial contents of the three blocks B, C and D, and the last quotient figure shows the thickness of those blocks, consequently, multiplying the triple square by the last quotient figure, gives the cubick contents of those blocks. Then the rule directs to "*multiply the square of the last quotient figure by the triple quotient.*" Now the triple quotient is the length of the blocks E, F and G, and the quotient figure shows their breadth, and their thickness; hence, multiplying the square of the last quotient figure by the triple quotient, gives the cubick contents of these blocks. Next we are directed to cube the last quotient figure. This cube shows the cubick contents of the block H, and the sum of these is equal to the cubick contents of the blocks B, C and D; and E, F and G.

2. What is the cube root of 34965783 ?

Ans. 327.

3. What is the cube root of 84·604519 ?

Ans. 4·39.

4. What is the cube root of ·008649 ?

Ans. ·2052+.

5. What is the cube root of $\frac{125}{343}$?

Ans. $\frac{5}{7}$.

APPLICATION AND USE OF THE CUBE ROOT.

Problem 1. *To find two mean proportionals between any two given numbers.*

Rule 1. Divide the greater number by the less, and extract the cube root of the quotient.

2. Multiply the root, so found, by the least of the given numbers, and the product will be the *less.*

3. Multiply this product by the same root, and it will give the *greater.*

EXAMPLES.

1. What are the two mean proportionals between 6 and 750?

$750 \div 6 = 125$, and $\sqrt[3]{125} = 5$. Then $5 \times 6 = 30$ the less, and $30 \times 5 = 150$ the greater. Ans. 30 and 150.

2. What are the two mean proportionals between 56 and 12096?
Ans. 336 and 2016.

Problem 2. *The side of a cube being given, to find the side of a cube which shall be any number of times greater or less than the given cube.*[*]

Rule.—Cube the given side, and if the required cube be greater than the given one, multiply the cube of the given side by the given proportion, and the cube root of the product will be the side of the cube required. But if the required cube be less, divide the cube of the given side by the given proportion, and the cube root of the quotient will be the side of the required cube.

EXAMPLES.

1. There is a cubick box whose side is 18 inches; what is the side of a box that will contain 8 times as much?

$18 \times 18 \times 18 = 5832$, and $5832 \times 8 = 46656$. Then $\sqrt[3]{46656} = 36$ in. Ans.

2. There is a cubick box whose side is 24 inches; what is the side of a box that will contain one sixty-fourth part as much?
Ans. 6 inches.

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[*] The solid, called a cube, has its length and breadth and height all equal. As the number of solid feet, inches, &c. in a cube are found by multiplying the height and length and breadth together; that is, by multiplying one side into itself twice, the third power of a number is called the *cube* of that number. The solid contents of similar figures are in proportion to each other, as the cubes of their similar sides or diameters.

PROBLEM 8. *The diameter of a globe or ball being given, to find the diameter of a globe or ball that shall be any number of times greater or less than the given one.*

RULE.—Cube the given diameter, and multiply or divide it by the given proportion as the question may require; and the cube root of the product, or quotient, will be the diameter of the globe required.

EXAMPLES.

1. If the diameter of a globe be 12 inches. what will be the diameter of a globe one eighth part as large? Ans. 6 inches.

2. If the diameter of a globe be 6 inches, what will be the diameter of a globe 64 times as large? Ans. 24 inches.

A GENERAL RULE FOR EXTRACTING THE ROOTS OF ALL POWERS.*

1. Prepare the given number for extraction, by pointing off from the units' place, as the required root directs.
2. Find the first figure of the root by trial, or by inspection into a table of powers, and subtract its power from the left hand period of the given number.
3. To the remainder bring down the first figure in the next period, and call it the dividend.
4. Involve the root to the next inferior power to that which is given, and multiply it by the number denoting the given power, for a divisor.
5. Find how many times the divisor may be had in the dividend, and the quotient will be another figure of the root.
6. Involve the whole root to the given power, and subtract it always from as many periods of the given number as you have found periods in the root.

---

* The roots of most powers may be found by the square and cube root only; therefore when any even power is given, the easiest method will be (especially in a very high power,) to extract the square root of it, which reduces it to half the given power; then the square root of that power reduces it to half the same power; and so on till you come to a square or cube.

For example; suppose a twelfth power be given; extracting the square root of the twelfth power reduces it to a sixth power; and extracting the square root of the sixth power reduces it to a cube.

296. *What is the general method of extracting roots of all powers?*

7. Bring down the first figure of the next period to the remainder for a new dividend, to which find a new divisor, as before, and in like manner proceed till the whole be finished.

EXAMPLES.

1. What is the cube root of 135796744?

135796744(514 the root, or answer.
5×5×5=125=1st subtrahend.

5×5×3=75)107=1st dividend.

51×51×51=132651=2nd subtrahend.

51×51×3=7803)31457=2nd dividend.

514×514×514=135796744=3d subtrahend.

2. What is the biquadrate or fourth root of 19987173376?
Ans. 376.

◆

ALLIGATION,

Alligation is the method of mixing two or more simples of different qualities, so that the composition may be of a mean or middle quality; it consists of two kinds, viz. Alligation Medial and Alligation Alternate.

ALLIGATION MEDIAL,

Is when the quantities and prices of several things are given, to find the mean price of the mixture compounded of those things.

RULE.

As the sum of the quantities, or the whole composition, is to the whole value; so is any part of the composition to its mean price or value.

---

297. *What is Alligation?*——298. *What is Alligation Medial?*——299. *What is the rule in Alligation Medial?*

EXAMPLES.

1. A grocer mixed 4 cwt. of sugar at $10 a cwt. with 5 cwt. at $9·50 a cwt. and 3 cwt. at $8·75 a cwt.—what was the value of one cwt. of this mixture ?

cwt.		$	$ c.
4	at	10	=40·00
5	"	9·50	=47·50
3	"	8·75	=26·25
cwt. 12			$113·75

cwt.		$ c.		cwt.
12	:	113·75	: :	1
		1		
12)113·75				
		$9·479$\frac{1}{6}$ Ans.		

2. If a bushel of Indian corn, at ·75 cents a bushel, be mixed with 5 bushels of rye at ·80 cents a bushel, and 15 bushels of oats at 30 cents a bushel ; what will be the value of a bushel of the mixture ?

Ans. ·44$\frac{1}{21}$ cents.

3. A wine merchant mixes 12 gallons of wine at ·75 cents a gallon, with 24 gallons at ·90 cents, and 16 gallons at $1·10 ; what is a gallon of this composition worth ?

c. m.
Ans. ·926$\frac{12}{13}$.

4. A goldsmith melted together 8oz. of gold of 22 carats fine,* 1 lb. 8oz. of 21 carats fine, and 10oz. of 18 carats fine ; what is the fineness of the composition.

Ans. 20$\frac{8}{19}$ carats fine.

ALLIGATION ALTERNATE

Is the method of finding what quantity of each of the ingredients, whose rates are given, will compose a mixture of a given rate : So that it is the reverse of Alligation Medial, and may be proved by it.

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* If an ounce or any other quantity of pure gold be divided into 24 equal parts, these parts are called carats, but gold is often mixed with some baser metal, which is called the alloy, and the mixture is said to be so many carats fine, according to the proportion of pure gold contained in it ; thus, if 22 carats of pure gold and 12 of alloy be mixed together, it is said to be 22 carats fine.——*See page* 43.

300. *What is Alligation Alternate ?*

CASE I.

When the mean rate of the whole mixture, and the rates of all the ingredients are given, and the quantity not limited.

RULE.

1. Place the several rates or prices of the simples, being reduced to one denomination, in a column under each other, and the mean price reduced to the same denomination, at the left hand.

2. Connect with a continued line the price of each simple or ingredient, which is *less* than that of the mean rate, with one, or any number of those, which are *greater* than the mean rate; and each *greater* rate or price with one, or any number of the *less*.

3. Place the difference between the mean price (or mixture rate) and that of each of the simples, opposite to the rates with which they are connected.

4. Then, if only one difference stand against any rate, it will be the quantity belonging to that rate; but if there be several, their sum will be the quantity.*

EXAMPLES.

1. A merchant has spices, some at 1s. 6d. a lb. some at 2s. some at 4s. and some at 5s. a lb.: How much of each sort must he mix, that he may sell the mixture at 3s. 4d. a lb.?

---

* By connecting the less rate to the greater, and placing the difference between them and the mean rate alternately, the quantities resulting are such, that there is precisely as much gained by one quantity as is lost by the other, and therefore the gain and loss upon the whole are equal, and are exactly the proposed rate.

In like manner, let the number of simples be what it may, and with how many soever each one is linked, since it is always a less with a greater than the mean price, there will be an equal balance of loss and gain between every two, and consequently an equal balance on the whole.

If any of the simples be of little or no value with respect to the rest, its rate is supposed to be nothing; as water mixed with wine, and alloy with gold or silver.

---

301. *How do you find the quantity of each ingredient, when the mean rate of the whole mixture, and the rates of all the ingredients are given, and the quantity not limited?*

	d.	lb.		s.	d.	
Mean rate 40d.	18	20	at	1	6	Ans.
	24	8	"	2	0	
	48	16	"	4	0	
	60	22	"	5	0	

	d.	lb.		s.	d.	
Mean rate 40d.	18	8	at	1	6	Ans.
	24	20	"	2	0	
	48	22	"	4	0	
	60	16	"	5	0	

NOTE. Questions in this case admit of as many answers, as there are various ways of connecting the rates of the ingredients together.

2. A grocer would mix the following qualities of sugar; viz. at 10 cents, 13 cents, and 16 cents a pound; what quantity of each sort must be taken to make a mixture worth 12 cents a pound?

Ans. 5 lbs. at 10 cts.
2 lbs. at 13 cts.
2 lbs. at 16 cts.

3. It is required to mix rum at 80 cents, and at 70 cents a gallon with water, that the mixture may be worth 75 cents a gallon; what quantity of each must be taken?

Ans. 80 galls. at 80 cts.
5 galls. at 70 cts.
5 galls. of water.

4. A goldsmith would mix gold of 19 carats fine, with some of 16, 18, 23, and 24 carats fine, so that the compound may be 21 carats fine; what quantity of each must he take?

Ans. 5 oz. of 16 carats fine.
5 oz. of 18 carats fine.
5 oz. of 19 carats fine.
10 oz. of 23 carats fine.
10 oz. of 24 carats fine.

CASE II.

When the rates of all the ingredients, the quantity of but one of them, and the mean rate of the whole mixture are given, to find the several quantities of the rest, in proportion to the quantity given.

RULE.

Take the difference between each price, and the mean rate, and place them alternately as in Case I. Then, as the difference standing against that simple, whose quantity is given, is to that quantity, so is each of the other differences, severally, to the several quantities required.

EXAMPLES.

1. A merchant has 20 lbs. of tea at $1 04 a pound, which he would mix with some at 98 cents, some at 92 cents, and some at 80 cents a pound; how much of each sort must he take to mix with the 20 pounds, that he may sell the mixture at 96 cents a pound?

$$96\left\{\begin{array}{l}104\\98\\92\\80\end{array}\right.\qquad\begin{array}{r}4\\16\\8\\2\end{array}$$

4 stand against the given quantity.

$$\text{As } 4 : 20 :: \left\{\begin{array}{rcrcr}\text{lbs.} & & \text{lbs.} & & \text{cts.}\\ 16 & : & 80 & \text{at} & 78\\ 8 & : & 40 & \text{“} & 92\\ 2 & : & 10 & \text{“} & 80\end{array}\right\}\text{Ans.}$$

2. Bought a pipe of brandy containing 120 gallons, at 1·30 a gallon; how much water must be mixed with it to reduce the first cost to $1·10 a gallon?

Ans. $21\frac{9}{11}$ galls.

3. How much gold of 16, 20 and 24 carats fine, and how much alloy, must be mixed with 10 oz. of 18 carats fine, that the composition may be 22 carats fine?

$$\text{Ans.}\left\{\begin{array}{l}\text{10 oz. of 16 carats fine.}\\ \text{10 oz. of 20 carats fine.}\\ \text{170 oz. of 24 carats fine.}\\ \text{10 oz. of alloy.}\end{array}\right.$$

302. *When the rates of all the ingredients, the quantity of but one of them, and the mean rate of the whole mixture are given; what is the rule for finding the several quantities of the rest, in proportion to the given quantity?*

CASE III.

When the rates of the several ingredients, the quantity to be compounded, and the mean rate of the whole mixture are given, to find how much of each sort will make up the quantity.

RULE.

Write the difference between the mean rate, and the several prices alternately, as in Case I; then, as the sum of the quantities or differences thus determined, is to the given quantity, or whole composition; so is the difference of each rate, to the required quantity of each rate.

EXAMPLES.

1. A merchant having sugars at 12 dollars, 10 dollars, and 8 dollars a cwt. would make a mixture of 30 cwt. worth 9 dollars a cwt.; what quantity of each must be taken?

$$9\left\{\begin{array}{l}12\\10\\8\end{array}\right. \quad \begin{array}{r}1\\1\\1+3=4\\\hline \text{sum}=6\end{array} \quad \text{As } 6 : 30 :: \left\{\begin{array}{rcrcr} \text{cwt.} & & \text{dols.} & & \\ 1 & : & 5 & \text{at} & 12 \\ 1 & : & 5 & \text{``} & 10 \\ 4 & : & 20 & \text{``} & 8 \\ \hline & & 30 & & \end{array}\right\} \text{Ans.}$$

2. A goldsmith has several sorts of gold; viz. of 15, 17, 20 and 22 carats fine, and would melt together of all these sorts, so much as may make a mass of 40 oz. of 18 carats fine; how much of each sort is required?

Ans. { 16 oz. of 15 carats fine.
8 oz. of 17 carats fine.
4 oz. of 20 carats fine.
12 oz. of 22 carats fine.

* * *

POSITION.

Position is a rule, by which any *true* or *required* numbers are found by means of *assumed* or *supposed* numbers. It is of two kinds, Single and Double.

---

303. When the rates of the several ingredients, the quantity to be compounded, and the mean rate of the whole mixture are given; what is the rule for finding how much of each sort will make up the quantity?—304. What is Position?

SINGLE POSITION

Is the working of one supposed number, as if it were the true one, to find the true number.

RULE.

1. Take any number and perform the same operations with it as are described to be performed in the question.

2. Then say as the sum of the errors is to the given sum, so is the supposed number to the true one required.*

PROOF. Add the several parts of the sum together, and if it agree with the sum, it is right.

EXAMPLES.

1. A school master, being asked how many scholars he had, said, if I had as many more as I now have, three quarters as many, half as many, one fourth and one eighth as many, I should then have 435:— Of what number did his school consist?

Suppose he had 80	As 290 : 435 : : 80	
As many=80	80	
$\frac{3}{4}$ as many=60	——	120
$\frac{1}{2}$ as many=40	29|0)3480|0(120 Ans.	120
$\frac{1}{4}$ as many=20	29	90
$\frac{1}{8}$ as many=10	—	60
—	58	30
290	58	15
	—	—
	0	435 Proof.

2. A person lent his friend a sum of money unknown, to receive interest for the same at 6 per cent. per annum, simple interest, and at the end of 12 years, received for principal and interest $860: What was the sum lent?

Ans. $500.

---

* The operations contained in the question being performed upon the answer or number to be found, will give the result contained in the question. The same operations, performed on any other number, will give a certain result. When the results are proportional to their supposed numbers, it is manifest that the result of the operations performed on the supposed number, must be to the supposed number, as the result in the question is to the true number or answser. In any case, when the results are not proportional to their supposed numbers, the answer cannot be found by this rule.

---

305. *What is Single Position?*——306. *What is the rule for working questions in Single Position?*

3. A, B, and C joined their stocks, and gained $353 12½c. of which A took up a certain sum, B took up four times as much as A, and C, five times as much as B: What share of the gain had each?

Ans.	$14	12½c.	A's share.
	56	50	B's share.
	282	50	C's share.

4. A and B, talking of their ages, B said his age was once and an half the age of A; C said his was twice and one tenth the age of both, and that the sum of their ages was 93: what was the age of each?
Ans. A's 12, B's 18, and C's 63 years.

5. Seven eights of a certain number exceeds four fifths by 6: what is the number? Ans. 80.

6. What number is that, which, being increased by $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{5}{6}$ of itself, the sum will be $234\frac{3}{4}$? Ans. 90

◆

DOUBLE POSITION.

Double Position teaches to resolve questions by making two suppositions of some convenient numbers.

Those questions, in which the results are not proportional, to their positions, belong to this rule: such are those, in which the number sought is increased or diminished by some given number, which is no known part of the number required.

RULE.*

1. Take any two convenient numbers, and proceed with each according to the conditions of the question.

2. Place the result or errours against their positions or supposed numbers, thus,

$$\begin{array}{ccc} \text{Pos.} & & \text{Err.} \\ 30 & & 12 \\ & \times & \\ 20 & & 6 \end{array}$$

and if the errour be too great, mark it with +; and if too small, with —.

---

* The rule is founded on this supposition, that the first error is to the second, as the difference between the true and first supposed number is to the difference between the true and second supposed number: When that is not the case, the exact answer to the question cannot be found by this rule.

307. *What is Double Position?*——308. *And what is the rule?*

3. Multiply them crosswise; that is, the first position by the last errour, and the last position by the first errour.

4. If the errours be alike, that is, both too small or both too great, divide the difference of the products by the difference of the errours, and the quotient will be the answer.

5. If the errours be unlike; that is, one too small, and the other too great, divide the sum of the products by the sum of the errours, and the quotient will be the answer.

Note.—When the errours are the same in quantity, and unlike in quality, half the sum of the suppositions is the number sought.

EXAMPLES.

1. A lady bought damask for a gown, at 8s. per yard, and lining for it at 3s. per yard; the gown and lining contained 15 yds., and the price of the whole was £3 10s.: How many yards were there of each?

Suppose 6 yards damask, value 48s.
Then she must have 9 yards lining, value 27s.

Sum of their values=75s.
So that the first errour is 5 too much, or + 5.

Again, suppose she had 4 yards of damask, value 32s.
Then she must have 11 yards of lining, value 33s.

Sum of their values=65s.
So that the second errour is 5 too little, or — 5s.

6 5+ £ s. d.
Then X 5 yards at 8s. = 2 0 0
5 5— 10 yards at 3s. = 1 10 0

20 30 3 10 0 proof.
20

Sum of errors=5+5=10)50

Ans. 5 yds. damask, and 15—5=10 yards [lining.
Or, 6+4÷2=5 as before.

2. A labourer was hired for 60 days upon this condition, that for every day he wrought, he should receive 75c.; and for every day he was idle, should forfeit 37½c.; at the expiration of the time he received $18: How many days did he work, and how many was he idle?

Ans. He was employed 36 days, and was idle 24.

3. There is a fish, whose head is 10 feet long; his tail is as long as his head and half the length of his body, and his body as long as the head and tail: What is the whole length of the fish?

Ans. 80 feet.

4. A farmer, having driven his cattle to market, received for them all $320, being paid at the rate of $24 per ox, $16 per cow, and $6 per calf: there were as many oxen as cows, and 4 times as many calves as cows: How many were there of each sort?

Ans. 5 oxen, 5 cows, and 20 calves.

PERMUTATION.

Permutation is the method of finding how many different ways the order or position of any given number of things may be changed or varied.

To find the number of permutations or changes that can be made of any given number of things, all different from each other.

RULE.

Multiply all the terms of the natural series of numbers, from one up to the given number, continually together, and the last product will be the answer required.*

---

* The reason of this rule may be shown thus, any one thing a is capable of one position only as, a.

Any two things a and b are capable of two variations only; as ab, ba; whose number is expressed by 1×2.

If there be three things a, b, and c; then any two of them, leaving out the third, will have 1×2 variations; and consequently, when the third is taken in, there will be 1×2×3 variations; and so on as far as you please.

---

309. *What is Permutation?*—310. *What is the rule for finding the number of changes that can be made of any given number of things, all different from each other?*

EXAMPLES.

1. How many changes or variations can be made of the three first letters of the alphabet?

$1 \times 2 \times 3 = 6$ Ans.

Changes or variations at large.

1	a	b	c
2	a	c	b
3	b	a	c
4	b	c	a
5	c	b	a
6	c	a	b

2. Christ church in Boston, has 8 bells: how many changes may be rung on them?

Ans. 40320.

3. Nine gentlemen met at an inn, and were so well pleased with their host, and with each other, that in a frolick, they agreed to tarry so long as they, together with their host, could sit every day in a different position at dinner: Pray how long, had they kept their agreement, would their frolick have lasted?

Ans. 3628800 days, $=9935\frac{55}{487}$ years.

GAUGING.

Gauging is the art of measuring all kinds of casks or vessels used for liquor, and of determining the quantity they will contain.

The instruments used in gauging are the gauging rod, callipers, the sliding rule, and Gunter's scale.

RULE.

Take the dimensions of the cask in inches, viz. the diameter at the bung and head, and the length of the cask; subtract the head diameter from the bung diameter, and note the difference.

If the staves of the cask be much curved or bulging between the bung and head, multiply the difference between the bung and head diameter by ·7; if not quite so much curved, by ·65; if they curve yet less, by ·6; and if they are almost or quite straight, by ·55, and add the product to the head diameter, the sum will be a mean diameter, by which the cask is reduced to a cylinder.

---

311. *What is Gauging?* — 312. *What are the rules for measuring casks, and finding their contents in gallons?*

Square the mean diameter, thus found, then multiply it by the length; divide the product by 294 for wine, or by 359 for ale or beer, and the quotient will be the content in gallons.

Note 1.—These divisors are found by dividing 231 and 282 by .7854.—If the square of the mean diameter be multiplied by .7854 and the product multiplied by the length, the last product will be the content in cubic inches, which being divided by 231 for wine, or by 282 for ale or beer, the quotient will be the content in gallons.

Note 2.—The length and head diameter are usually taken by callipers, allowing for the thickness of both heads, 1 inch, 1½ inch, or 2 inches, according to the size of the cask.

The head diameter must be taken close to its outside, and for small casks, add 3 tenths of an inch; for casks of 30, 40, or 50 gallons, 4 tenths; and for larger casks 5 or 6 tenths; and the sum will be very nearly the head diameter within.

The bung diameter is usually taken by the gauging rod, and in taking it, observe by moving the rod backward and forward, whether the stave, opposite the bung, be thicker or thinner than the rest, and if it be, make allowance accordingly.

EXAMPLE.

What is the content in wine, and ale or beer gallons, of a cask, whose bung diameter is 35 inches, head diameter 27 inches, and length 45 inches?

Bung diameter=35
Head diameter=27

Difference=8
.7

5.5
Add the head diam. 27

32.6
32.6

1956
652
978

Squared 1062.76

Square of the diameter=1062.76
Length= 45

531380
425104

47824.20

47824.20÷294=162.69+ Wine galls.
47824.20÷359=133.21+ Ale or Beer galls.
Ans.

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313. *How do you obtain the divisors named in the rule?*——314. *How are the different diameters of the cask to be ascertained?*

MECHANICAL POWERS.

OF THE LEVER OR STEELYARD.

It is a principle in mechanics, that the power is to the weight, as the velocity of the weight, is to the velocity of the power.

Therefore, *to find what weight may be raised or balanced by any given power*, say ;

As the distance between the body to be raised or balanced, and the *fulcrum or prop*, is to the distance between the prop and the point where the power is applied ; so is the power to the weight which it will balance.

If a man weighing 160 lbs. rest on the end of a lever 10 feet long, what weight will he balance on the other end ; supposing the prop one foot from the weight.

The distance between the weight and the prop being 1 foot, the distance from the prop to the power is 10—1=9 feet ; therefore, as 1 foot : 9 feet : : 160 lbs. : 1440 lbs. Ans.

In giving directions for making a chaise, the length of the shafts between the axletree and backband, being settled at 9 feet, a dispute arose whereabout on the shafts the centre of the body should be fixed. The chaisemaker advised to place it 30 inches before the axletree ; others supposed 20 inches would be a sufficient incumbrance for the horse : Now supposing two passengers to weigh 336 pounds, and the body of the chaise 84 pounds more ; what will the beast in both these cases bear more than his harness ?

Weight of the passengers and chaise=420 lbs. and 9 feet=108 inches :

$$\text{Then, as 108 in. : 420 lbs.}\begin{array}{ccc} & \text{in.} & & \text{lbs.} & \\ \left\{\vphantom{\begin{array}{c}1\\1\end{array}}\right. & \begin{array}{c}30\\20\end{array} & \begin{array}{c}:\\:\end{array} & \begin{array}{r}116\frac{2}{3}\\77\frac{7}{9}\end{array} & \left.\vphantom{\begin{array}{c}1\\1\end{array}}\right\}\end{array}\text{Ans.}$$

OF THE WHEEL AND AXLE.

The proportion for the wheel and axle (in which the power is applied to the circumference of the wheel, and the weight is raised by a rope, which coils about the axle as the wheel turns round) is as the diameter of the axle is to the diameter of the wheel, so is the power applied to the wheel, to the weight suspended by the axle.

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315. *What is a general principle in mechanics?*——316. *What is the power of the lever or steelyard, or how do you find what weight may be raised or balanced by any given power?* ——317. *What is the power of the wheel and axle, or how do you find what weight suspended from the axle, will be balanced by any given power applied to the wheel?*

If the diameter of the axle be 6 inches, and the diameter of the wheel, 60 inches; what weight suspended from the axle will be balanced by a power of 1 lb. applied to the wheel?

in. in. lb. lbs.

As 6 : 60 : : 1 : 10 Ans.

OF THE SCREW.

The power is to the weight which is to be raised, as the distance between two threads of the screw, is to the circumference of a circle described by the power applied to the end of the lever.

RULE.

Find the circumference of the circle described by the end of the lever; then, as that circumference is to the distance between the spiral threads of the screw; so is the weight to be raised, to the power which will raise it, abating the friction which is not proportional to the quantity of surface, but to the weight of the incumbent part; and, at a medium, ⅓ part of the effect of the machine is destroyed by it, sometimes more and sometimes less.

There is a screw whose threads are an inch asunder; the lever by which it is turned 30 inches long, and the weight to be raised a ton, or 2240 lbs.: What power or force must be applied to the end of the lever, sufficient to turn the screw—that is, to raise the weight?

The lever being the semi-diameter of the circle, the diameter is 60 inches; then, as 113 : 355 : : 60 : 188.496 inches nearly, the circumference.

in. in. lbs. lbs.

Therefore, 188.496 : 1 : 2240 : 11.88+ Ans.

Let the weight be 2240lb. the power 11.88lb. and the lever 30 inches; Required the distance between the threads?

lbs. lbs. in. in.

As 2240 : 11.88 : : 288.496 : 1 nearly, Ans.

Let the power be 11.88lb., the weight 2240lb., and the threads an inch asunder, to find the length of the lever.

lbs. lbs. in. in.

As 11.88 : 2240 : : 1 : 188.5; then, as 355 : 113 : : 188.5 : 60 inches nearly, the diameter, and 60÷2=30 inches, Ans.

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318. *What is the power of the screw?*——319. *What is the rule for finding the power which must be applied to the end of the lever, sufficient to turn the screw, that is, to raise any given weight?*——320. *When required, how do you find the distance between the spiral threads of the screw; or the length of the lever?*

USEFUL AND DIVERTING EXERCISES.

1. What difference is there between twice 5 and twenty, and twice twenty-five? Ans. 20.

2. In an orchard of fruit trees $\frac{1}{2}$ of them bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plums, 60 of them peaches, and 40 cherries; how many trees does the orchard contain? Ans. 1200.

3. A merchant begins the world with $5000, and finds that by his distillery he clears $5000, in 6 years; by his navigation, $5000 in 7$\frac{1}{2}$ years; and that he spends in gambling $5000 in 3 years; how long will his estate last? Ans. 30 years.

4. A. can do a piece of work alone in 7 days, and B. in 12; in what time will both, working together, finish it? Ans. 4$\frac{8}{19}$ days.

5. A. and B. are on opposite sides of a circular field 268 poles about; they being to go round it, both the same way at the same instant of time; A. goes 22 rods in 2 minutes, and B. 34 rods in 3 minutes; how many times will they go round the field before the swifter overtakes the slower?
Ans. A. 16$\frac{1}{2}$ times.
B. 17 times.

6. A water tub holds 73 gallons; the pipe, which conveys the water to it usually admits 7 gallons in 5 minutes; and the tap discharges 20 gallons in 17 minutes. Now, supposing both these to be carelessly left open, and the water to be turned on at 4 o'clock in the morning: a servant, at 6, finding the water running, puts in the tap: in what time, after this accident, will the tub be filled?
Ans. 32 min. 58$\frac{118}{119}$ sec.

7. A hare starts 12 rods before a hound; but is not perceived by him till she has been up 45 seconds; she scuds away at the rate of 10 miles an hour; and the dog, on view, makes after her at the rate of 16 miles an hour; how long will the course hold, and what space will be run over, from the spot where the dog started?
Ans. 2288 ft. 97$\frac{1}{2}$ seconds.

8. Required the number, from which, if 7 be subtracted, and the remainder be divided by 8, and the quotient be multiplied by 5, and 4 added to the product, the square root of the sum extracted, and three fourths of that root cubed, the cube, divided by 9, the last quotient may be 24? Ans. 103.

9. Suppose a lighthouse built on the top of a rock; the distance between the place of observation and that part of the rock level with the eye, 620 yards; the distance from the top of the rock to the place of observation, 846 yards; and from the top of the light house 900 yards; the height of the light house is required?
Ans. 76.77+ yds.

10. Sound, uninterrupted, moves at the rate of 1142 feet per second; if the time between the lightning and thunder be one minute, at what distance was the explosion? Ans 12.977+ miles.

12. If the earth be 7911 miles in diameter, and the moon 2180 miles; how many moons will it take to make one earth?

Ans. 47.788

12. A father left his estate of $1300 per annum to his only son, but he being only 14 years of age, his guardian was to pay $100 per annum for board, education, &c., and the surplus was to be put out to interest for his benefit, at 6 per cent. compound interest; now allowing no loss, what sum had his guardian to pay him when he was of age?

Ans. 1115 dols. 33 cts. 5 m.

13. Hiero, king of Sicily, ordered his jeweller to make a crown, containing 63 ounces of gold. The workman thought that substituting part silver was only a proper perquisite, which taking air, Archimedes was appointed to examine it, who on putting it into a vessel of water found it raised the fluid 8.2245 cubick inches, and having discovered that the inch of gold weighed 10.36 ounces, and that of silver but 5.85 ounces, he found what part of the king's gold had been changed. Repeat the process and inform us what part of it was gold, and what silver?

Ans. 28.8038 oz. silver.
34.1962 oz. gold.

14. A person having driven a stock of cattle to market, received for them all 456 dollars, he was paid at 50 dollars for each horse, 20 dollars for each cow, and 4 dollars for each sheep; the number of cows was double the number of horses; and there were three times as many sheep as cows: what did he receive for the horses, what for the cows, and what for the sheep, and how many of each sort were there.

Ans.	24 Sheep	$ 96
	8 Cows	$160
	4 Horses	$200

15. A gate-keeper is to receive 6 cents for every wagon, 4 cents for every gig, 2 cents for every horseman, and 1 cent for every footman that passes the gate; at the year's end he found that 3150 gigs had passed, and that 7 gigs passed when 5 wagons did, and 4 horsemen passed when 6 footmen did, and 5 footmen passed when 3 gigs did, what number of wagons, horsemen, and footmen passed, and how much did the gate-keeper receive?

Ans.	Gigs	3150
	Wagons	2250
	Footmen	5250
	Horsemen	3500
	Amount of toll	$353.50

16. If 1000 bricks lie 6 inches from each other in a straight line, and a person be employed to gather them up one by one, and place them on a pile which is one foot from the first brick, how far will he have walked when he shall have placed the last brick on the pile?

Ans. 94m. 7fur. 186yds. 2ft.

17. A man dying left his wife in expectation that a child would be afterwards added to the surviving family; and making his will, ordered, that, if the child were a son, $\frac{2}{3}$ of his estate should belong to him,

and the remainder to his mother; but, if it were a daughter, he appointed the mother $\frac{2}{3}$, and the child the remainder. But it happened, that the addition was both a son and a daughter, by which the widow lost in equity, $2400 more than if there had been only a girl. What would have been her dowry had she had only a son?

Ans. $2100.

18. When first the marriage knot was tied
Betwixt my wife and me,
My age with hers did so agree,
As nineteen does with eight and three;
But after ten and half ten years,
We man and wife had been,
Her age came up so near to mine,
As two times three to nine.
What were our ages at marriage?

Ans. 57 and 33.

19. Three jealous husbands with their wives, being ready to pass by night over a river, do find at the water side a boat which can carry but two persons at once, and for want of a waterman, they are necessitated to row themselves over the river at several times: The question is, how those six persons shall pass by 2 and 2, so that none of the three wives may be found in the company of one or two men, unless her husband be present?

20. As I was going to St. Ives,
I met seven wives,
Every wife had seven sacks,
Every sack had seven cats,
Every cat had seven kits,
Kits, cats, sacks and wives,
How many were going to St. Ives?

21. A countryman having a Fox, a Goose, and a peck of corn, in his journey, came to a river, where it so happened that he could carry but one over at a time. Now as no two were to be left together that might destroy each other; so he was at his wit's end how to dispose of them; for, says he, though the corn can't eat the goose, nor the goose eat the fox; yet the fox can eat the goose, and the goose eat the corn. The question is, how he must carry them over that they may not devour each other?

22. A man driving his geese to market, was met by another, who said, good morrow master, with your hundred geese; says he, I have not an hundred, but if I had half as many as I now have, and two geese and a half beside the number I now have already, I should have an hundred. How many had he?

23. Two men were driving sheep to market, says one to the other, give me one of yours and I shall have as many as you: the other says, give me one of yours and I shall have as many again as you. How many had each?

[illegible]

BOOK-KEEPING.

[After the scholar, male or female, has acquired a competent knowledge of Arithmetick, or of its fundamental rules, instruction in the mode of *keeping accounts* should be attended to. By this it is not meant to recommend that the son or daughter of every farmer, mechanick, or shop keeper should enter deeply into the science as practised by the merchant, engaged in extensive business, for such study would engross a great portion of time which might be more usefully employed in acquiring a proper knowledge of a trade or other employment.

Persons employed in the common business of life, who do not keep regular accounts, are subjected to many losses and inconveniences; to avoid which, the following simple and correct plan is recommended for their adoption.

Let a small book be made, or a few sheets of paper sewed together, and ruled after the examples given in this system. In the book, termed the Day-Book, are duly to be entered, daily, all the transactions of the master or mistress of the family, which require a charge to be made, or a credit to be given to any person. No article thus subject to be entered, should on any consideration, be deferred till another day. Great attention should be given to write the transaction in a plain hand; the entry should mention all the particulars necessary to make it fully understood with the time when they took place; and if an article be delivered, the name of the person to whom delivered is to be mentioned. No scratching out may be suffered; because it is sometimes done for dishonest purposes, and will weaken or destroy the authority of your accounts. But if, through mistake, any transaction should be wrongly entered, the error must be rectified, by a new entry; and the wrong one may be cancelled by writing the word *Error*, in the margin.

A book, thus fairly kept, will at all times show the exact state of a person's affairs, and have great weight, should there at any time be a necessity of producing it in a court of justice.

The instructer, who feels a parental solicitude for the permanent welfare of his pupils, cannot in any way so much contribute to their success in life, with so little trouble, as to teach them to understand this abridged, complete and simple system of Book-Keeping. It contains all the important principles of extended and expensive works on the science; all, in fact, that is necessary to be known by the Farmer, Mechanick and Shopkeeper, relating to accounts; and yet with very little explanation and repeated copying and balancing the accounts, will be so fully understood and deeply impressed on the memory of scholars of common mind, as never to be forgotton; while their knowledge of common arithmetick and practical penmanship will thereby be greatly improved.]

BOOK-KEEPING is the art of recording mercantile transactions in a regular and systematick manner.

Book-Keeping by SINGLE ENTRY chiefly records the transactions on credit, and for this purpose two books are necessary, called the Day-Book, and the Leger.

THE DAY-BOOK.

Each page of the Day-Book should be ruled with two columns on the right hand for dollars and cents, and one column on the left for inserting the page or folio of the Leger on which the account is posted.

The Day-Book begins with an account of the owner's property, debts, &c.; then follows a detail of the occurrences of trade, set down in the order of time in which they take place.

116
90
40
21.67
70
185

306.67

39
26

30.0
29

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